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Analyzing Paths in Time Petri Nets

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Abstract. In this paper, a parametric description of a transition sequence in a Time Petri net is introduced. The minimal and maximal time duration of a transition sequence are shown to be integers and furthermore the min/max path passes only integer-states. A necessary condition for the reachability of an arbitrary state is given.

Keywords: Time Petri Net, integer state, reachability of a state, parametric description of a transition sequence

1. Introduction

One of the most adequate ways of modeling and analyzing concurrent systems with an infinite state space is given by the theory of Petri nets. For studying systems in which local time dependencies between actions are relevant, a large variety of time dependent Petri nets have been defined and widely applied. For some of them, analyzing methods have emerged,

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whereas for others there are at most simulation tools. One comprehensive yet convenient variant of such Petri nets are the Time Petri nets (TPNs) introduced by Merlin [6] and studied by Berthomieu/Menasche [4], Berthomieu/Diaz [3], Popova [7], [8] Berthelot/Boucheneb [2], Aalst [1] etc.

TPNs are classical Petri nets in which to each transition t a time interval $[a_t, b_t]$ is assigned additionally, where a_t and b_t are relative to the time when t was last enabled. When t becomes enabled, it can not fire until a_t time units have elapsed, and it must fire no later than b_t, unless t is disabled by the firing of another transition. Firing a transition takes no time. Thus, the basic idea of this kind of time dependent Petri nets may be described as follows: when a transition becomes enabled, it must (in general) not fire at once but only during a certain time interval, and at the end of that interval, firing is compulsory. However, if the interval is not bounded above, then the obligation to fire becomes obsolete.

For analyzing Time Petri nets there are two different methods: one has been defined by Berthomieu/Diaz/Menasche and the other by Popova. They both compute a reachability graph of the TPN, but the two methods rely on different concepts.

In Berthomieu/Diaz/Menasche, the notion "state" is used to describe (infinitely) many situations in Time Petri nets: a state is defined as a marking together with time intervals for the enabled transitions (cf. Berthomieu/ Menasche [4], Berthomieu/Diaz [3]), while in Popova [7] a state is a description of a snapshot of the net. Here, a state is given by a marking together with a time vector consisting of the current local time for the enabled transitions and a special symbol ($\sharp$) for the disabled transitions.

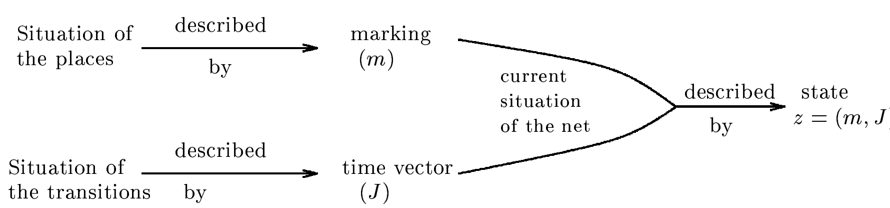

Figure 1. *Meaning and structure of a state according to Popova [7]*

Consequently, two entirely different methods for constructing a reachability graph for a TPN exist: a node of the Berthomieu/Diaz/Menasche's reachability graph (a state class (cf. Berthomieu/Menasche [4] , Berthomieu/Diaz [3]) includes an infinite number of possible situations of the TPN and is given as a solution of a system of inequalities.

The nodes of the reachability graph as defined in Popova [7]) are only the "essential" states. These are integer-states, i.e. states when the current local time for each enabled transition is an integer. For example, $z = (\underbrace{(1,0,3)}_{marking}, \underbrace{(3,\sharp,0,\sharp)}_{timevec.})$ is a state in a Time Petri net with 3 places and 4 transitions. The first and the third transitions are enabled. The first transition has been enabled for 3 time units, and the third transition has just become enabled. The second and the fourth transitions are disabled.

Which integer-states are "essential"? Generally, for a reachable integer-state, all integral components of the time vector are bounded above by the latest firing time of the corresponding transition. However, there may be transitions with infinite latest firing time. For these transitions the earliest firing time can be used, instead, as an upper bound for the corresponding component (cf. Popova [9]). These further restrictions characterize the set of essential states within the set of reachable integer-states.

Obviously, this reachability graph includes only a discrete part of all situations which are possible for a given TPN. Nevertheless, the knowledge of the net behaviour in the "essential" states is sufficient to determine the entire behavior of the net at **every** time (cf. Popova [7], Popova [9]). While the calculation of a single integer-state is easy, both ways of computing the state space can lead to a state explosion.

In this paper, an alternative method for investigating fundamental properties of firing sequences with respect to time will be developed, which does not make use of the computation of the whole state space.

The following section consists of some preliminary remarks. In the third section the central concept of parametric sequences is introduced, and important extremal characteristics are proved. The final section provides a brief overview of applications for which implementations of this method already exist.

2. Basic Notations and Definitions

The following notations are used: $\mathbb{N}$ is the set of natural numbers, $\mathbb{N}^+ := \mathbb{N} \setminus \{0\}$. $\mathbb{Q}_0^+$ is the set of nonnegative rational numbers. Let g be a given function from A to B. T^* denotes the language of all words over the alphabet T, including the empty word e; $l(w)$ is the length of the word w. $\mathfrak{P}(C)$ denodes the power set of a set C. C^D is the cartesian product $\underbrace{C \times \cdots \times C}_{card(D)\ times}$.

Definition 2.1. The structure $N = (P, T, F, V, m_o)$ is called a Petri net (PN) iff
(1) P, T, F are finite sets with
$P \cap T = \emptyset$, $\quad P \cup T \neq \emptyset$, $\quad F \subseteq (P \times T) \cup (T \times P)$ and
$dom(F) \cup cod(F) = P \cup T$
(2) $V : F \longrightarrow \mathbb{N}^+$ (weight of the arcs)
(3) $m_o : P \longrightarrow \mathbb{N}$ (initial marking)

A marking of a PN is a function $m : P \longrightarrow \mathbb{N}$, such that $m(p)$ denotes the number of tokens at the place p. The pre- and post-sets of a transition t are given by $Ft := \{p \mid p \in P \wedge pFt\}$ and $tF := \{p \mid p \in P \wedge tFp\}$, respectively. Each transition $t \in T$ induces the marking t^- and

t^+, defined as follows:

$$t^-(p) = \begin{cases} V(p,t) & ,\text{iff} \quad (p,t) \in F \\ 0 & ,\text{iff} \quad (p,t) \notin F \end{cases} \qquad t^+(p) = \begin{cases} V(t,p) & ,\text{iff} \quad (t,p) \in F \\ 0 & ,\text{iff} \quad (t,p) \notin F \end{cases}$$

Moreover, Δt denotes $t^+ - t^-$. A transition $t \in T$ is enabled (may fire) at a marking m iff $t^- \leq m$ (e.g. $t^-(p) \leq m(p)$ for every place $p \in P$). When an enabled transition t at a marking m fires, this yields a new marking m' given by $m'(p) := m'(p) + \Delta t(p)$ and denoted by $m \xrightarrow{t} m'$.

Definition 2.2. The structure $Z = (P, T, F, V, m_o, I)$ is called a Time Petri Net (TPN) iff:
(1) $S(Z) = (P, T, F, V, m_o)$ is a PN.
(2) $I : T \longrightarrow \mathbb{Q}_0^+ \times (\mathbb{Q}_0^+ \cup \{\infty\})$ and $I_1(t) \leq I_2(t)$ for each $t \in T$, where $I(t) = (I_1(t), I_2(t))$.
A TPN is called finite Time Petri net (FTPN) iff $I : T \longrightarrow \mathbb{Q}_0^+ \times \mathbb{Q}_0^+$.

I is the time function of Z, $I_1(t)$ and $I_2(t)$ the earliest firing time of t ($eft(t)$) and the latest firing time of t ($lft(t)$), respectively. It is not difficult to see (cf. Popova[8]) that considering TPNs with $I : T \longrightarrow \mathbb{N} \times (\mathbb{N} \cup \{\infty\})$ will not result in a loss of generality. Therefore only such time functions I will be considered subsequently. Furthermore, *conflict* is used in the strong sense: two transitions t_1 and t_2 are in conflict iff $Ft_1 \cap Ft_2 \neq \emptyset$. The PN $S(Z)$ referred to as the skeleton of Z.

Within this approach, the definition of a state is of fundamental importance for the ensuing theory. A state is characterized by a marking together with the momentary local time for enabled transitions or the sign $\sharp$ for the disabled transitions.

Definition 2.3. Let $Z = (P, T, F, V, m_o, I)$ be a TPN and $J : T \longrightarrow \mathbb{Q}_0^+ \cup \{\#\}$. $z = (m, J)$ is called a state in Z iff:
(1) m is a reachable marking in $S(Z)$.
(2) $\forall t \ (\ (t \in T \wedge t^- \leq m) \ \longrightarrow \ J(t) \leq lft(t))$.
(3) $\forall t \ (\ (t \in T \wedge t^- \not\leq m) \ \longrightarrow \ J(t) = \#)$.

Interpretation of the notion "state" is as follows: within the net, each transition t has a clock. If t is enabled at a marking m, the clock of t shows the time elapsed since t became most recently enabled. If t is disabled at m, the clock does not work (indicated by $\#$).

Now the dynamic aspects of TPNs – changing from one state into another – can be introduced: The state $z_o := (m_o, J_o)$ with $J_o(t) := \begin{cases} 0 & \text{iff} \quad t \leq m_0 \\ \# & \text{iff} \quad t \not\leq m_0 \end{cases}$ is set as the initial state of the TPN Z. A transition t is ready to fire in the state $z = (m, J)$, denoted by $z \xrightarrow{t}$, iff $t^- \leq m$ and $eft(t) \leq J(t)$. A transition $\hat{t}$, which is ready to fire in the state $z = (m, J)$, may fire yielding a new state $z' = (m', J')$, defined by $m' = m + \Delta\hat{t}$ and

$$J'(t) =: \begin{cases} \# & \text{iff} \quad t^- \not\leq m \\ J(t) & \text{iff} \quad t^- \leq m \wedge t^- \leq m' \wedge Ft \cap F\hat{t} = \emptyset \\ 0 & \text{otherwise} \end{cases} .$$

The state $z = (m, J)$ is changed into the state $z' = (m', J')$ by the time duration $\tau \in \mathbb{Q}_0^+$, denoted by $z \xrightarrow{\tau} z'$, iff $m' = m$ and the time duration τ is possible (formally: $\forall t\ (\ (t \in T \wedge J(t) \neq \#) \longrightarrow J(t) + \tau \leq lft(t)$ and $J'(t) := \begin{cases} J(t) + \tau & \text{iff} \quad t \leq m \\ \# & \text{iff} \quad t \not\leq m \end{cases}$). $z = (m, J)$ is called an integer-state iff $J(t)$ is an integer for each enabled transition t in m. The graph $RG_Z(z_o)$ is called the reachability graph of the TPN Z iff its nodes are the reachable integer-states and its arcs are defined by the triples (z, t, z') and (z, τ, z'), where $z \xrightarrow{t} z'$ and $z \xrightarrow{\tau} z'$, respectively. Let $R_Z(z)$ be the set of all reachable states in a TPN Z when started in z. A transition t is called live in z iff $\forall z'(z' \in R_Z(z) \longrightarrow \exists z''(z'' \in R_Z(z') \wedge t$ is ready to fire in $z''))$. The state z is live in Z iff all transitions $t \in T$ are live in z, and Z is live iff z_o is live in Z.

3. Parametric description of transition sequences

In order to study transition sequences in a TPN, the notion of a **parametric description** of a sequence will be introduced. Consider the following net:

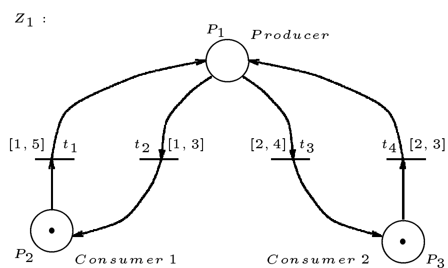

Figure 2. *A feasible sequence in a TPN*

The initial state is $z_0 = ((0, 1, 1), (0, \sharp, \sharp, 0))$. In the initial state the transitions t_1 and t_4 are enabled, but neither t_1 nor t_4 may fire because of their time restrictions. Thus, z_o can change into another state only as time elapses. For example, the change of states $z_o \xrightarrow{1.7} z_1$ is feasible, where z_1 is given by $m_1 = m_0$ and $J_1 = (1.7, \sharp, \sharp, 1.7)$. Furthermore, z_1 can change into the state z_2 with $z_1 \xrightarrow{1.0} z_2$, where the state z_2 is given by $m_2 = m_1$ and $J_2 = (2.7, \sharp, \sharp, 2.7)$. In z_2 the transition t_4 can fire, yielding the state z_3 with: $m_3 = m_2 + \Delta t_4 = (1, 1, 0)$ and $J_3 = (2.7, 0, 0, \sharp)$. Now, as time progresses to $\tau_3 = 2$, state z_3 changes into the state z_4, with $m_4 = m_3$, $J_4 = (4.7, 2.0, 2.0, \sharp)$. Subsequently, t_1 fires and z_4 is changes into a state z_5 with $m_5 = (2, 0, 0)$ and $J_5 = (\sharp, 2.0, 2.0, \sharp)$. The initial state and the state z_5 are integer-states whereas the states z_1, z_2, z_3 and z_4 are not.

The resulting sequence $z_o \xrightarrow{1.7} z_1 \xrightarrow{1.0} z_2 \xrightarrow{t_4} z_3 \xrightarrow{2.0} z_4 \xrightarrow{t_1} z_5$ is feasible in Z_1. The transition sequence $\sigma = (t_4 t_1)$ (or the transition-time sequence $\theta = (\tau_1 \tau_2 t_4 \tau_3 t_1)$, where $\tau_1 = 1.7, \tau_2 = 1.0$, and $\tau_3 = 2.0$) is executable in Z_1.

Obviously, a transition sequence in a TPN depends on time: each transition fires at a certain moment in a certain time interval. Therefore, a transition sequence in a Time Petri Net is, in fact, an alternating sequence of time durations and transitions (e.g., $\tau_0 t_0 \tau_1 t_1 ...$, which means: at the beginning the time τ_0 is elapsed. After that the transition t_0 fires. Then time τ_1 elapses and afterwards the transition t_1 fires, etc.).

In order to investigate the time dependencies of transition sequences, the time durations elapsing between two adjacent transitions of the sequence will be given by parameters which satisfy suitable inequalities. It is convenient to transfer the subject matter into the terminology of an elementary predicate calculus: Let $[F, P, K]$ be a signature with symbols for functions, predicates and constants defined as $F := \{F^2\}$, $P := \{A^2\}$, and $K := \{u_t, v_t | t \in T\} \cup \{\$\}$, respectively. Taking $D := \mathbb{Q}_0^+ \cup \{\sharp\}$ as a domain for the set of variables $X := \{x_i \mid i \in \mathbb{N}\}$, the interpretation $[D, \omega]$ will be considered, with ω given by:

$$\omega(F^2) := +, \quad \omega(A^2) := \leq, \quad \omega(u_t) := \text{eft}(t), \quad \omega(v_t) := \text{lft}(t), \quad \omega(\$) := \sharp.$$

Here, $+$ is a binary operation on D, which coincides with the well-known addition in $\mathbb{Q}_0^+$. In this context, it is not necessary to specify $+$ any further. Similar considerations apply to $\leq$.

Let SUM be the union of the set of all terms, in which each variable appears at most once and constants do not appear at all, and the set which consists only of the constant \$. COND denotes the set of all formulae $A^2 term_i term_j$ where $term_i \in \text{SUM} \setminus \{\$\}$ and $term_j \in K \setminus \{\$\}$, or vice versa; the term in $\{term_i, term_j\} \cap \text{SUM} \setminus \{\$\}$ is denoted by $s(A^2 term_i term_j)$, the one in $\{term_i, term_j\} \cap K \setminus \{\$\}$ with $r(A^2 term_i term_j)$. Under this interpretation, the value of a *term* with respect to an evaluation β will be denoted by $|term|_\beta$.

Definition 3.1. Let $Z = [P, T, F, V, m_0, I]$ be a TPN. The function $\delta : T^* \longrightarrow R_Z(m_0) \times \text{SUM}^T \times \mathfrak{P}(\text{COND})$ is partially defined by induction:

Basis: $\delta(e) := [m_e, \Sigma_e, B_e]$ where

1. $m_e = m_0$
2. $\Sigma_e(t) := \begin{cases} x_0 & \text{iff } t^- \leq m_e \\ \$ & \text{otherwise} \end{cases}$
3. $B_e := \{A^2 \Sigma_e(t) v_t | t^- \leq m_e\}$.

Step: Let σ be a transition sequence and assume that $\delta(\sigma)$ has been defined as $[m_\sigma, \Sigma_\sigma, B_\sigma]$. For a transition $\hat{t}$ with $\Sigma_\sigma(\hat{t}) \neq \$$, $\delta(\sigma\hat{t}) = [m_{\sigma\hat{t}}, \Sigma_{\sigma\hat{t}}, B_{\sigma\hat{t}}]$ is defined as follows:

1. $m_{\sigma\hat{t}} := m_\sigma + \Delta(\hat{t})$,
2. $\Sigma_{\sigma\hat{t}}(t) := \begin{cases} \$ & \text{iff } t^- \not\leq m_{\sigma\hat{t}} \\ x_{l(\sigma)+1} & \text{iff } (t^- \not\leq m_{\sigma t} \wedge t^- \leq m_{\sigma\hat{t}}) \vee \\ & (t^- \leq m_{\sigma t} \wedge t^- \leq m_{\sigma\hat{t}} \wedge Ft \cap F\hat{t} \neq \emptyset) \\ F^2(\Sigma_\sigma(t), x_{l(\sigma)+1}) & \text{otherwise} \end{cases}$

3. $B_{\sigma\hat{t}} := B_{\sigma t} \cup \{A^2 u_t \Sigma_\sigma(\hat{t})\} \cup \{A^2 \Sigma_{\sigma\hat{t}}(t) v_t \mid t^- \leq m_{\sigma\hat{t}}\}$

With regard to the interpretation of the symbols for functions, predicates and constants in the logic defined above, the following notational conventions for terms in $SUM \backslash \{\$\}$, formulae in *COND*, and constants will be used for reasons of convenience and increased readability:

$$x_1 + \cdots + x_n := F^2(\ldots(F^2(x_1, x_2), \ldots, x_n),$$
$$term_1 \leq term_2 := A^2(term_1, term_2),$$
and instead of constant symbols their interpretation under ω.

The set of all states, C_σ which can be reached in Z from z_0 under the restrictions that σ has been fired, can be defined inductively as follows: $C_0 := \{z \mid \exists\tau(\tau \in \mathbb{Q}_0^+ \wedge z_0 \xrightarrow{\tau} z)\}$. Assuming that C_σ has been defined, then, for a transition $\hat{t}$, $C_{\sigma\hat{t}} := \{z \mid \exists z_1 \exists z_2 \exists \tau(z_1 \in C_\sigma \wedge \tau \in \mathbb{Q}_0^+ \wedge z_1 \xrightarrow{\hat{t}} z_2 \xrightarrow{\tau} z)\}$.

Apparently, there is a close connection between these state classes and the mapping δ defined above: $C_\sigma = \{(m_\sigma, \Sigma_\sigma(t)) \mid B_\sigma\}$.

Example 3.1.

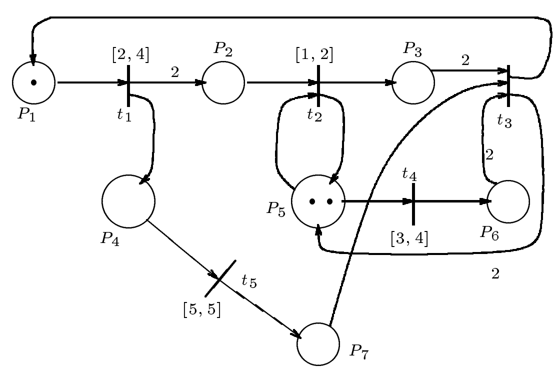

Figure 3. A simple example illustrating state classes

$$C_e = \{(1,0,0,0,2,0,0), \begin{pmatrix} x_1 \\ \sharp \\ \sharp \\ x_0 \\ \sharp \end{pmatrix} \mid \begin{array}{c} 0 \leq x_0 \\ x_0 \leq 4 \end{array} \}.$$

$$C_{t_1} = \{(0,2,0,1,2,0,0), \begin{pmatrix} \sharp \\ x_1 \\ \sharp \\ x_0 + x_1 \\ x_1 \end{pmatrix} \mid \begin{array}{l} 2 \leq x_0 \\ x_0 \leq 4 \\ 0 \leq x_1 \\ x_1 \leq 2 \\ 0 \leq x_0 + x_1 \\ x_0 + x_1 \leq 4 \end{array} \}, \ldots$$

$$C_{t_1 t_4 t_2 t_5} = \{(0,1,1,0,1,1,1), \begin{pmatrix} \sharp \\ x_3 + x_4 \\ \sharp \\ x_3 + x_4 \\ \sharp \end{pmatrix} \mid \begin{array}{l} 2 \leq x_0,\ x_0 \leq 4,\ 0 \leq x_0 + x_1,\ x_0 + x_1 \leq \\ 2,\ 0 \leq x_1,\ x_1 \leq 2,\ 3 \leq x_0 + x_1,\ x_1 + x_2 \leq \\ 5,\ x_2 \leq 2,\ 1 \leq x_2,\ x_1 + x_2 + x_3 = 5,\ 0 \leq \\ x_3,\ x_3 \leq 2,\ 0 \leq x_3 + x_4 \leq 2 \end{array} \}.$$

etc...

The next two properties follow easily from the definitions.

Proposition 3.1. *Let Z be a TPN, $\sigma \in T^*, C_0 \xrightarrow{\sigma} C_\sigma$ and $\delta(\sigma) = [m_\sigma, \Sigma_\sigma, B_\sigma]$. Then, for each state $z \in C$, there is a evaluation $\beta : X \longrightarrow \mathbb{Q}_0^+$ such that: $z = (m_\sigma, \mid \Sigma_\sigma \mid_\beta)$ and $\bigwedge_{b \in B_\sigma} \beta$ satisfies b.*

Proposition 3.2. *(Converse of 3.1)*
Let Z be a TPN, $\sigma \in T^*, C_0 \xrightarrow{\sigma} C_\sigma$ and $\delta(\sigma) = [m_\sigma, \Sigma_\sigma, B_\sigma]$). For each evaluation $\beta : X \longrightarrow \mathbb{Q}_0^+$ with $\bigwedge_{b \in B_\sigma} \beta$ satisfies b, the state $z := (m_\sigma, |\Sigma_\sigma|_\beta)$ is in C_σ.

The following two properties can be proved by induction on the transition sequence σ:

Proposition 3.3. *Let σ be a sequence in Z and $\delta(\sigma) = [m_\sigma, \Sigma_\sigma, B_\sigma]$ be its parametric description. Then, for any two transitions t_i and t_j in Z with $\Sigma_\sigma(x_i) = x_{i_0} + x_{i_1} + \cdots + x_{i_k}$ and $\Sigma_\sigma(x_j) = x_{j_0} + x_{j_1} + \cdots + x_{j_l}$, it follows that $i_{k-r} = j_{l-r}$ for all $r = 1, \ldots, \min\{k, l\}$.*

Proposition 3.4. *Let σ be a sequence in Z and $\delta(\sigma) = [m_\sigma, \Sigma_\sigma, B_\sigma]$ be its parametric description. Then,*

1. *for each transition $t \in T$ it is true that: if $\Sigma_\sigma(t) = x_i + \ldots + x_j$ then each variable x_k with $i \leq k \leq j$ also appears in $\Sigma_\sigma(t)$.*
2. *for each* term $\in$ SUM , *which is a part of a formula b in B_σ, it is true that: if* term $= x_i + \ldots + x_j$ *then each variable x_k with $i \leq k \leq j$ also appears in* term.

Now, a central characteristic for transition sequences in TPNs can be shown.

Theorem 3.1. *Let $Z = [P, T, F, V, m_0, I]$ be a TPN, σ a transition sequence of length n, with $\delta(\sigma) = [m_\sigma, \Sigma_\sigma, B_\sigma]$ and $\hat{\beta} : X \longrightarrow \mathbb{Q}_0^+$ an evaluation such that $\forall c(c \in B_\sigma \rightarrow \hat{\beta}$ satisfies $c)$. Then there exists an evaluation $\beta^* : X \longrightarrow \mathbb{N}$ such that:*

1. $\forall c(c \in B_\sigma \rightarrow \beta^*$ *satisfies* $c)$
2. $\forall t(t \in T \wedge t^- \leq m_\sigma \rightarrow |\Sigma_\sigma(t)|_{\beta^*} \leq |\Sigma_\sigma(t)|_{\hat{\beta}})$
3. $\big|\sum\limits_{k=0}^{n} x_k\big|_{\beta^*} \leq \big|\sum\limits_{k=0}^{n} x_k\big|_{\hat{\beta}}$

Proof:

Idea

An evaluation β^* with the required properties will be explicitly constructed out of the given evaluation $\hat{\beta}$ by subsequently transforming each non-integral rational to a nearby integer.

As default value, the maximum of the set of integers which are not greater than the given rational (i.e. the "floor" of that rational denoted by $\lfloor r \rfloor$), will be taken, in order to ensure that the second and third property stated in the theorem are satisfied. By doing so, the restriction yielded by the first property will be somewhat loosened, i.e. temporarily, it is sufficient that a required condition is "almost" satisfied. This means, that for each formula c in B_σ, the value of the non-constant term $s(c)$ under the current evaluation will only have to lie in a certain neighbourhood of the initial value $|s(c)|_{\hat{\beta}}$.

If, by taking the integer part of the rational value for a certain variable, such a neighbourhood will be left for at least one condition, the minimum of the set of integers which are not smaller than the given rational r (i.e. the "ceiling" of that rational denoted by $\lceil r \rceil$), will be taken instead. The largest part of the proof aims to show that the three requirements stated above will also be satisfied (with the first one once again "loosened") in this case.

To complete the proof, it then remains to verify, that for the finally constructed evaluation, which takes only integer values, the "loosened" version of the first requirement is equivalent to the original one.

Construction of β^*

Let X_σ be the set of all variables which appear in B_σ, i.e. $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ Define a finite sequence of evaluations $\beta_i : X_\sigma \longrightarrow \mathbb{Q}_0^+$ by induction:

Basis: $\beta_0 : X_\sigma \longrightarrow \mathbb{Q}_0^+$ with $\beta_0(x) := \hat{\beta}(x)$ for all $x \in X_\sigma$.

Step: Assume that β_{i-1} has been defined. In order to describe the construction of β_i, the following function is used:

$$\underline{\beta_i}(x) := \begin{cases} \beta_{i-1}(x) & \text{iff } x \neq x_{n-i+1} \\ \lfloor \beta_{i-1}(x) \rfloor & \text{otherwise} \end{cases}$$

Now define $\beta_i : X_\sigma \longrightarrow \mathbb{Q}_0^+$ by

$$\beta_i(x) := \begin{cases} \beta_{i-1}(x) & \text{iff } x \neq x_{n-i+1} \\ \lfloor \beta_{i-1}(x) \rfloor & \text{iff } x = x_{n-i+1} \wedge \forall c(c \in B_\sigma \rightarrow \lfloor |s(c)|_{\beta_0} \rfloor - 1 < |s(c)|_{\underline{\beta_i}}) \\ \lceil \beta_{i-1}(x) \rceil & \text{otherwise} \end{cases}$$

In words, $\underline{\beta_i}$ describes the way, how the value of the variable x_{n-i+1} currently considered should be modified in the default case. However, if for any condition c in B_σ, the value of the term $s(c)$ is decreased below the bound $\lfloor |s(c)|_{\beta_0} \rfloor - 1$ as a consequence, x_{n-i+1} will be set to $\lceil \beta_{i-1}(x) \rceil$ instead.

Note that in each step the value of exactly one variable is changed, and that the value changed in a specific step is not altered by other steps before or afterwards. This implies in particular, that for variables x_k with $k < n - j + 1$

$$\beta_j(x_k) = \beta_{j-1}(x_k) = \ldots = \beta_0(x_k) \tag{1}$$

and that for variables x_k with $k \geq n - j + 1$

$$\beta_j(x_k) = \beta_{j+1}(x_k) = \ldots = \beta_n(x_k) \tag{2}$$

Furthermore, if $\beta_0(x_{n-i+1})$ is already an integer, then β_i leaves x_{n-i+1} unaltered, since for any integer k, $k = \lfloor k \rfloor = \lceil k \rceil$.

The following three assertions about this sequence are proved by induction on i:

1. $\forall i \forall c (c \in B_\sigma \to |s(c)|_{\beta_i} \in (\lfloor |s(c)|_{\beta_0} \rfloor - 1, \lceil |s(c)|_{\beta_0} \rceil + 1)$
2. $\forall i \forall t (t \in T \wedge t^- \leq m_\sigma \to |\Sigma_\sigma(t)|_{\beta_i} \leq |\Sigma_\sigma(t)|_{\beta_0})$
3. $|\sum_{k=0}^{n} x_k|_{\beta_i} \leq |\sum_{k=0}^{n} x_k|_{\beta_0}$

Basis: For $i = 0$, all three assertions are trivially true.

Step: Assume that the assertion has been justified for each of $1, \ldots, i$, and consider the case $i + 1$. If $\beta_i(x_{n-i}) \in \mathbb{N}$, then $\beta_{i+1} = \beta_i$ and all assertions follow immediately from the induction hypothesis. Therefore, it may be assumed that $\beta_i(x_{n-i})$ is not an integer.

Let b be any condition in B_σ. If x_{n-i} does not appear in $s(b)$, then $|s(b)|_{\beta_{i+1}} = |s(b)|_{\beta_i}$, and the first assertion follows from the induction hypothesis. Hence, assume that x_{n-i} is in $s(b)$.

Two cases need to be considered:

Case 1: $\beta_{i+1}(x_{n-i}) = \lfloor \beta_i(x_{n-i}) \rfloor$

Since $\beta_{i+1}(x) \leq \beta_i(x)$ for each x, it is evident that

$$|\sum_{k=0}^{n} x_k|_{\beta_{i+1}} \leq |\sum_{k=0}^{n} x_k|_{\beta_i}$$

Using the induction hypothesis for i, this proves the third assertion in the case $i + 1$.

Similarly, for a transition t which is not disabled after the firing of σ, the inequality

$$|\Sigma_\sigma(t)|_{\beta_{i+1}} \leq |\Sigma_\sigma(t)|_{\beta_i}$$

combined with the induction hypothesis proves the second assertion.

Furthermore, the fact that $\forall x(\beta_{i+1}(x) \leq \beta_i(x)$ can also be used to give

$$|s(b)|_{\beta_{i+1}} \leq |s(b)|_{\beta_i}$$

By the induction hypothesis, $|s(b)|_{\beta_i} < \lceil |s(b)|_{\beta_0} \rceil + 1$, so the previous inequality becomes

$$|s(b)|_{\beta_{i+1}} < \lceil |s(b)|_{\beta_0} \rceil + 1 \tag{3}$$

As $\beta_{i+1}(x_{n-i})$ has been set to $\lfloor \beta_i(x_{n-i}) \rfloor$, the corresponding criteria in the definition of β_{i+1}

$$\forall c(c \in B_\sigma \rightarrow \lfloor |s(c)|_{\beta_0} \rfloor - 1 < |s(c)|_{\underline{\beta_{i+1}}})$$

has been fulfilled. Since $\underline{\beta_{i+1}} = \beta_{i+1}$, this gives for the condition b in particular:

$$\lfloor |s(b)|_{\beta_0} \rfloor - 1 < |s(b)|_{\beta_{i+1}} \tag{4}$$

Because b was chosen arbitrarily, the inequalities (3) and (4) combined prove the first assertion in the case $i + 1$, and therefore complete the induction step.

Case 2: $\beta_{i+1}(x_{n-i}) = \lceil \beta_i(x_{n-i}) \rceil$

Since $\beta_{i+1}(x) \geq \beta_i(x)$ for each x, it is evident that

$$|s(b)|_{\beta_{i+1}} \geq |s(b)|_{\beta_i}$$

By the induction hypothesis, $|s(b)|_{\beta_i} > \lfloor |s(b)|_{\beta_0} \rfloor - 1$, so the previous inequality becomes

$$|s(b)|_{\beta_{i+1}} > \lfloor |s(b)|_{\beta_0} \rfloor - 1 \tag{5}$$

As $\beta_{i+1}(x_{n-i})$ has not been set to $\lfloor \beta_i(x_{n-i}) \rfloor$, the corresponding criteria in the definition of β_{i+1}

$$\forall c(c \in B_\sigma \rightarrow \lfloor |s(c)|_{\beta_0} \rfloor - 1 < |s(c)|_{\underline{\beta_{i+1}}})$$

has not been fulfilled.

Therefore, a condition $\tilde{c}$ exists in B_σ, such that

$$|s(\tilde{c})|_{\underline{\beta_{i+1}}} \leq \lfloor |s(\tilde{c})|_{\beta_0} \rfloor - 1 \tag{6}$$

If x_{n-i} did not appear in $\tilde{c}$, then by definition $\underline{\beta_{i+1}}$ would be identical to β_i on all variables which appear in $\tilde{c}$, which would imply that $|s(\tilde{c})|_{\underline{\beta_{i+1}}} = |s(\tilde{c})|_{\beta_i}$. But then (6) yields a contradiction to the induction hypothesis. Hence, x_{n-i} does appear in $\tilde{c}$. By the definition of $\underline{\beta_{i+1}}$, this gives

$$|s(\tilde{c})|_{\underline{\beta_{i+1}}} = |s(\tilde{c})|_{\beta_i} - \beta_i(x_{n-i}) + \lfloor \beta_i(x_{n-i}) \rfloor \tag{7}$$

Substituting (7) into (6) gives

$$|s(\tilde{c})|_{\beta_i} - \beta_i(x_{n-i}) + \lfloor \beta_i(x_{n-i}) \rfloor \leq \lfloor |s(\tilde{c})|_{\beta_0} \rfloor - 1$$

On noting that $\lceil \beta_i(x_{n-i}) \rceil = \lfloor \beta_i(x_{n-i}) \rfloor + 1$, the last inequality may be rearranged to give

$$|s(\tilde{c})|_{\beta_i} - \beta_i(x_{n-i}) + \lceil \beta_i(x_{n-i}) \rceil \leq \lfloor |s(\tilde{c})|_{\beta_0} \rfloor \tag{8}$$

But since $\beta_{i+1}(x_{n-i}) = \lceil \beta_i(x_{n-i}) \rceil$, the left hand side of (8) is just $|s(\tilde{c})|_{\beta_{i+1}}$, and therefore

$$|s(\tilde{c})|_{\beta_{i+1}} \leq \lfloor |s(\tilde{c})|_{\beta_0} \rfloor \tag{9}$$

which in particular implies

$$|s(\tilde{c})|_{\beta_{i+1}} < |s(\tilde{c})|_{\beta_0} \tag{10}$$

Now suppose that

$$|s(b)|_{\beta_{i+1}} \geq \lceil |s(b)|_{\beta_0} \rceil + 1 \tag{11}$$

which in particular implies

$$|s(b)|_{\beta_{i+1}} > |s(b)|_{\beta_0} + 1 \tag{12}$$

Let $j_{\tilde{c}}$ and $k_{\tilde{c}}$ be the minimal and maximal variable index which appears in $s(\tilde{c})$, respectively. By Proposition 3.4 above,

$$\omega(s(\tilde{c})) = x_{j_{\tilde{c}}} + x_{j_{\tilde{c}}+1} + \ldots + x_{n-i} + x_{n-i+1} + \ldots + x_{k_{\tilde{c}}} \tag{13}$$

Similarly, there is are indices j_b and k_b such that

$$\omega(s(b)) = x_{j_b} + x_{j_b+1} + \ldots + x_{n-i} + x_{n-i+1} + \ldots + x_{k_b} \tag{14}$$

Hence (10) may be rewritten as

$$\begin{aligned}
&(\beta_{i+1}(x_{j_{\tilde{c}}}) - \beta_0(x_{j_{\tilde{c}}})) + \\
&(\beta_{i+1}(x_{j_{\tilde{c}}+1}) - \beta_0(x_{j_{\tilde{c}}+1})) + \ldots + \\
&(\beta_{i+1}(x_{n-i}) - \beta_0(x_{n-i})) + (\beta_{i+1}(x_{n-i+1}) - \beta_0(x_{n-i+1})) + \ldots + \\
&(\beta_{i+1}(x_{k_{\tilde{c}}}) - \beta_0(x_{k_{\tilde{c}}})) < 0
\end{aligned} \tag{15}$$

Setting $j := i + 1$ in (1) gives

$$\beta_{i+1}(x_k) = \beta_i(x_k) = \ldots \beta_0(x_k) \quad \text{for all } k < n - i \tag{16}$$

Therefore (15) can be simplified as

$$
\begin{aligned}
&(\beta_{i+1}(x_{n-i}) - \beta_0(x_{n-i}) + (\beta_{i+1}(x_{n-i+1}) - \beta_0(x_{n-i+1})) + \ldots + \\
&(\beta_{i+1}(x_{k_{\tilde{c}}}) - \beta_0(x_{k_{\tilde{c}}})) < 0
\end{aligned}
\tag{17}
$$

Similarly, (12) and (14) give

$$
\begin{aligned}
&(\beta_{i+1}(x_{n-i}) - \beta_0(x_{n-i})) + (\beta_{i+1}(x_{n-i+1}) - \beta_0(x_{n-i+1})) + \ldots + \\
&(\beta_{i+1}(x_{k_b}) - \beta_0(x_{k_b})) > 1
\end{aligned}
\tag{18}
$$

Three sub-cases need to be considered:

Case 2.1: $k_{\tilde{c}} = k_b$

Then the left hand sides of the inequalities (17) and 18 are identical, yielding the contradiction $1 < 0$.

Case 2.2: $k_{\tilde{c}} < k_b$

Setting $j := n - k_{\tilde{c}}$ in 1 gives

$$\beta_0(x_k) = \beta_{n-k_{\tilde{c}}}(x_k) \quad \text{for all } k < k_{\tilde{c}} + 1,$$

so that inequality (17) becomes

$$
\begin{aligned}
&(\beta_{i+1}(x_{n-i}) - \beta_{n-k_{\tilde{c}}}(x_{n-i}) + \\
&(\beta_{i+1}(x_{n-i+1}) - \beta_{n-k_{\tilde{c}}}(x_{n-i+1})) + \ldots + \\
&(\beta_{i+1}(x_{k_{\tilde{c}}}) - \beta_{n-k_{\tilde{c}}}(x_{k_{\tilde{c}}})) < 0
\end{aligned}
\tag{19}
$$

By (16), β_{i+1} and $\beta_{n-k_{\tilde{c}}}$ agree on all variables with indices smaller than $n-i$. Setting $j := n - k_{\tilde{c}}$ in (2) shows that β_{i+1} and $\beta_{n-k_{\tilde{c}}}$ also agree on all variables with indices greater than $k_{\tilde{c}}$.

Hence, (19) together with (14) show that

$$|s(b)|_{\beta_{i+1}} - |s(b)|_{\beta_{n-k_{\tilde{c}}}} < 0 \tag{20}$$

But (11) and (20) then give

$$|s(b)|_{\beta_{n-k_{\tilde{c}}}} > \lceil |s(b)|_{\beta_0} \rceil + 1$$

which contradicts the induction hypothesis for $n - k_{\tilde{c}}$.

Case 2.3: $k_{\tilde{c}} > k_b$

Setting $j := n - k_b$ in (1) gives

$$\beta_0(x_k) = \beta_{n-k_b}(x_k) \quad \text{for all } k < k_b + 1,$$

so that inequality (18) becomes

$$
\begin{aligned}
&(\beta_{i+1}(x_{n-i}) - \beta_{n-k_b}(x_{n-i}) + \\
&(\beta_{i+1}(x_{n-i+1}) - \beta_{n-k_b}(x_{n-i+1})) + \ldots + \\
&(\beta_{i+1}(x_{k_b}) - \beta_{n-k_b}(x_{k_b})) > 1
\end{aligned}
\tag{21}
$$

By (16), β_{i+1} and β_{n-k_b} agree on all variables with indices smaller than $n-i$. Setting $j := n - k_b$ in (2) shows that β_{i+1} and β_{n-k_b} also agree on all variables with indices greater than k_b.

Hence, (21) together with (13) show that

$$|s(\tilde{c})|_{\beta_{i+1}} - |s(\tilde{c})|_{\beta_{n-k_b}} > 1 \tag{22}$$

But (9) and (22) then give

$$|s(\tilde{c})|_{\beta_{n-k_b}} < \lfloor |s(\tilde{c})|_{\beta_0} \rfloor - 1$$

which contradicts the induction hypothesis for $n - k_b$.

The assumption (11) has led to a contradiction in all three sub-cases. Therefore the following inequality must hold:

$$|s(b)|_{\beta_{i+1}} < \lceil |s(b)|_{\beta_0} \rceil + 1 \tag{23}$$

Since b was chosen arbitrarily, (5) and (23) prove the first assertion.

Setting $j := n - k_{\tilde{c}}$ in (1) gives

$$\beta_0(x_k) = \beta_{n-k_{\tilde{c}}}(x_k) \quad \text{for all } k < k_{\tilde{c}} + 1,$$

so that inequality (17) becomes

$$
\begin{aligned}
&(\beta_{i+1}(x_{n-i}) - \beta_{n-k_{\tilde{c}}}(x_{n-i}) + (\beta_{i+1}(x_{n-i+1}) - \beta_{n-k_{\tilde{c}}}(x_{n-i+1})) + \\
&\ldots + (\beta_{i+1}(x_{k_{\tilde{c}}}) - \beta_{n-k_{\tilde{c}}}(x_{k_{\tilde{c}}})) < 0
\end{aligned}
\tag{24}
$$

By (16), β_{i+1} and $\beta_{n-k_{\tilde{c}}}$ agree on all variables with indices smaller than $n - i$. Setting $j := n - k_{\tilde{c}}$ in (2) shows that β_{i+1} and $\beta_{n-k_{\tilde{c}}}$ also agree on all variables with indices greater than $k_{\tilde{c}}$.

Hence, (24) shows that

$$|\sum_{k=0}^{n} x_n|_{\beta_{i+1}} < |\sum_{k=0}^{n} x_n|_{\beta_{n-k_{\tilde{c}}}}$$

But by the induction hypothesis for $n - k_{\tilde{c}}$,

$$|\sum_{k=0}^{n} x_n|_{\beta_{n-k_{\tilde{c}}}} \leq |\sum_{k=0}^{n} x_n|_{\beta_0}$$

which together with the previous inequality proves the third assertion:

$$|\sum_{k=0}^{n} x_n|_{\beta_{i+1}} < |\sum_{k=0}^{n} x_n|_{\beta_0}$$

Let t be a transition which is not disabled after the firing of σ. If x_{n-i} does not appear in $\Sigma_\sigma(t)$, then $|\Sigma_\sigma(t)|_{\beta_{i+1}} = |\Sigma_\sigma(t)|_{\beta_i}$, and $|\Sigma_\sigma(t)|_{\beta_{i+1}} \leq |\Sigma_\sigma(t)|_{\beta_0}$ follows from the induction hypothesis. Therefore, assume that x_{n-i} does appear in $\Sigma_\sigma(t)$.

By the construction of Σ_σ (cf. 3.3), x_n appears in every component which is not \$. Referring to Proposition 3.4, this implies that there is an index j_t such that

$$\Sigma_\sigma(t) = x_{j_t} + x_{j_t+1} + \ldots + x_{n-i} + x_{n-i+1} + \ldots + x_n \tag{25}$$

By (16), β_{i+1} and $\beta_{n-k_{\tilde{c}}}$ agree on all variables with indices smaller than $n - i$. Setting $j := n - k_{\tilde{c}}$ in (2) shows that β_{i+1} and $\beta_{n-k_{\tilde{c}}}$ also agree on all variables with indices greater than $k_{\tilde{c}}$.

Hence, (19) together with (25) show that

$$|\Sigma_\sigma(t)|_{\beta_{i+1}} - |\Sigma_\sigma(t)|_{\beta_{n-k_{\tilde{c}}}} < 0 \tag{26}$$

Using the induction hypothesis for $n - k_{\tilde{c}}$, (26) gives

$$|\Sigma_\sigma(t)|_{\beta_{i+1}} \leq |\Sigma_\sigma(t)|_{\beta_0}$$

Since t was chosen arbitrarily, the second assertion is also proved, which completes the induction step.

Setting $\beta^*(x) := \beta_n(x)$ for all x in X_σ, a function $\beta^* := X \longrightarrow \mathbb{N}$ can be defined, which satisfies property 2 and 3 stated in the theorem, as the second and third assertion just proved hold for $i = n$ in particular.

Consider a condition c in B_σ. Since β^* assigns only integers to the variables, $|s(c)|_{\beta^*}$ is also an integer. The first assertion above implies that

$$|s(c)|_{\beta^*} \in (\lfloor |s(c)|_{\beta_0} \rfloor - 1, \lceil |s(c)|_{\beta_0} \rceil + 1)$$

But the only integers in the interval $(\lfloor |s(c)|_{\beta_0} \rfloor - 1, \lceil |s(c)|_{\beta_0} \rceil + 1)$ are $\lfloor |s(c)|_{\beta_0} \rfloor$ and $\lceil |s(c)|_{\beta_0} \rceil$.

$r(c)$ is a constant symbol, which is interpreted by ω as an integer, namely $\mathrm{eft}(t)$ or $\mathrm{lft}(t)$ for some transition t. Clearly, if for a given rational r and an integer i, the inequalities $r \leq i$ or $i \leq r$ hold, then $\lfloor r \rfloor \leq i$ or $i \leq \lfloor r \rfloor$ are also fulfilled, respectively, and the same applies to $\lceil r \rceil$.

Therefore, for both possible values $\lfloor |s(c)|_{\beta_0} \rfloor$ and $\lceil |s(c)|_{\beta_0} \rceil$ of $|s(c)|_{\beta^*}$, it follows that β^* satisfies c.

Since c was chosen arbitrarily, β^* satisfies all conditions in B_σ, so that property 1 stated in the theorem is also proved. ■

Analogously, the following theorem may be proved.

Theorem 3.2. *Let $Z = [P, T, F, V, m_0, I]$ be a TPN, σ a transition sequence of length n, with $\delta(\sigma) = [m_\sigma, \Sigma_\sigma, B_\sigma]$ and $\hat{\beta} : X \longrightarrow \mathbb{Q}_0^+$ an evaluation such that $\forall c(c \in B_\sigma \rightarrow \hat{\beta}$ satisfies $c)$. Then there exists an evaluation $\beta^* : X \longrightarrow \mathbb{N}$ such that:*

1. $\forall c(c \in B_\sigma \rightarrow \beta^*$ *satisfies* $c)$
2. $\forall t(t \in T \wedge t^- \leq m_\sigma \rightarrow |\Sigma_\sigma(t)|_{\beta^*} \geq |\Sigma_\sigma(t)|_{\hat{\beta}})$
3. $|\sum_{k=0}^{n} x_k|_{\beta^*} \geq |\sum_{k=0}^{n} x_k|_{\hat{\beta}}$

Corollary 3.1. *Let $z = (m, J)$ be an arbitrary reachable state in a TPN Z. Then the states $\underline{z} := (m, \lfloor J \rfloor)$ and $\overline{z} := (m, \lceil J \rceil)$ are also reachable in Z.*

Proof: The existence of $\underline{z} := (m, \lceil J \rceil)$ follows immediately from theorem 3.1 (2), and the existence of $\overline{z} := (m, \lceil J \rceil)$ – from theorem 3.2 (2). ■

4. Applications and final remarks

Time dependent predicates are indispensable for the verification of time sensitive systems. For example, in order to support dependability engineering by assuring the meeting of prescribed deadlines in a concurrent control system, minimal and maximal paths are requested with respect to time (cf. Popova/Heiner [5]).

In general, there is no relation between the behaviour of a TPN and its skeleton. Thus, choosing time intervals does not only determine the quantitative behaviour of the net, but may also be crucial for its qualitative aspects. For systems which are modeled with TPNs, an analysis can be done in order to improve their reliability (cf. Popova/Heiner [5]).

Moreover, the question arises: under which time restrictions is it possible that pre-determined qualitative and/or quantitative properties are fulfilled? Using the parametric description of a sequence, the following qualitative problems can be solved: in a given Petri Net time intervals can be computed for each transition such that

- a given transition sequence which is enabled in the PN remains enabled in the TPN
- a feasible branch in the PN becomes non-feasible in the TPN.

Obviously, the parametric description of a sequence leads to a system of inequalities. Consequently, the computation of the minimal and maximal duration of a sequence is a linear programming problem. According to the central theorems 3.1 and 3.2, the min/max duration of the path is an integer, and the states which the min/max path passes are integer-states.

Hence, the solution can be found using linear programming, or (if there is no state explosion) the reachability graph (whose nodes are integer-states!).

Using the parametric description of a sequence, the theorems 3.1 and 3.1, and linear programming techniques, the solution of the following problems with respect to a given transition sequence σ, are implemented in the tool t-INA (cf. Starke [10]).

1. assuring that the sequence can be executed
2. assuring that prescribed deadlines for the given sequence are met,
3. assuring that a prescribed minimal time duration of the sequence is observed,
4. checking whether a realization of σ with a prescribed time duration exists,
5. finding time restrictions which ensure that prescribed (dead) states in the system become unreachable.

The tool is written in C and it is running under UNIX.

Additionally, the corollary 3.1 provides a necessary condition for the reachability of an arbitrary state z: $\underline{z}$ and $\overline{z}$ have to be reachable, too. Hence, one of them is not reachable, the state z can also not be reached in Z.

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