Connectivity of Wireless Multihop Networks in a Shadow Fading Environment

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Abstract. This article analyzes the connectivity of multihop radio networks in a log-normal shadow fading environment. Assuming the nodes have equal transmission capabilities and are randomly distributed according to a homogeneous Poisson process, we give a tight lower bound for the minimum node density that is necessary to obtain an almost surely connected subnetwork on a bounded area of given size. We derive an explicit expression for this bound, compute it in a variety of scenarios, and verify its tightness by simulation. The numerical results can be used for the practical design and simulation of wireless sensor and ad hoc networks. In addition, they give insight into how fading affects the topology of multihop networks. It is explained why a high fading variance helps the network to become connected.

Keywords: Wireless multihop networks, wireless sensor networks, ad hoc networking, connectivity, node isolation.

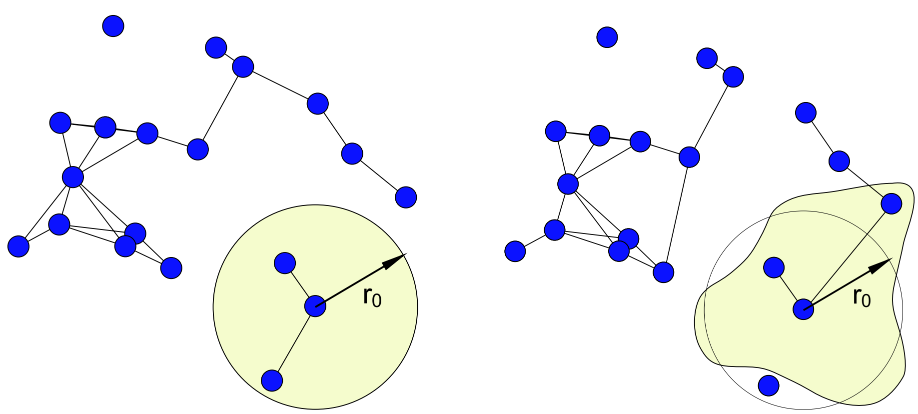
1. Introduction and Motivation

There has been a tremendous amount of research on self-organizing wireless multihop networks during the last few years. In such networks, the mobile devices communicate with each other in a peer-to-peer fashion without the need for base stations or any other pre-existing network infrastructure. If two devices cannot establish a direct wireless link — because they are too far away from each other — devices in between act as relays to forward the data from the source to the destination. In other words, each device acts as both a mobile terminal and a node of the network. The fact that multihop networks can be established "on the fly" gave them the name "ad hoc networks." This communication paradigm is especially useful to establish spontaneous networks among mobile computers, sensor networks for environmental monitoring, and car networks for telematics applications.

This article investigates the topology of such networks. The multihop paradigm creates a topology with characteristics that are quite different from those of a cellular network. This is especially apparent if we consider the *connectivity* among the devices. While a mobile device in a cellular system is "connected" if it has a wireless link to at least one base station, the situation in a wireless multihop network is more complicated, since each single mobile device contributes to the connectivity of the entire network. In fact, the level of connectivity among devices depends on their spatial density, transmission and reception capabilities, and the characteristics of the wireless channel.

One of the first papers related to connectivity issues in wireless multihop networks was [7]. It investigated how far a node's broadcast message percolates if the nodes are randomly distributed according to a homogeneous Poisson point process on an infinitely large area. Another early paper [22] addressed connectivity issues for nodes that are randomly distributed according to a uniform probability distribution on a one-dimensional line segment. More recently, Gupta and Kumar [15] performed a fundamental study on the connectivity of uniformly distributed nodes on a circular area. Further analytical investigations of the connectivity in bounded areas were made by Santi and Blough [26, 27], Bettstetter [2, 3], and Desai [9]. Dousse *et al.* [11] considered nodes on an infinite line and gave an expression for the probability that two nodes with a given distance can establish a multihop path between them.

All these papers study connectivity-related properties by employing a very simple model to characterize the wireless channel: two nodes are linked together, if and only if they are not further apart than a certain threshold distance r_0 (see Fig. 1 a). Such a purely geometric model is only sufficient as long as deterministic, distance-dependent channel models are considered. It is well-known, however, that the wireless channel can be modeled in a more realistic manner. Most important, one should consider its randomness induced by shadowing effects that are caused by obstacles. It was shown, for example, in [28, 29] that a more accurate modeling of the physical layer is indeed important in network-level research on wireless multihop networks.



a. Purely geometric link model b. Shadow fading link model

Figure 1. Illustration of link models and resulting network topologies

This observation is our motivation to investigate the network connectivity with consideration of shadow fading. We are interested in the question: How does shadow fading change the connectivity properties? Clearly, by considering fading, the distance between two nodes is no longer sufficient to determine whether these nodes establish a link or not. A node's transmission range, which was assumed to be a fixed parameter r_0 in previous research, becomes a stochastic parameter. Hence, the connectivity analysis becomes more complex.

Our contributions are as follows: Section 2 explains the used network model, including the channel model with shadowing. Section 3 addresses connectivity from the local viewpoint of a node: it analyzes the number of neighbors that a node has; in particular, it gives the probability that a node is isolated $P\,(\mathrm{iso})$, i.e., the node has no wireless link to any other node. Section 4 includes the main results of this article. It studies the overall connectivity of a subnetwork on a given subregion. We derive a tight bound for the critical node density that is needed to achieve, with high probability $P(\mathrm{con}) = 99\,\%$, a connected subnetwork. In addition to the theoretical derivation, we compute the critical density in a variety of practical scenarios and interpret the results. We observe that fading has significant impact on connectivity properties. The analysis is accompanied by simulation results that validate the theory and show that the derived bound is very tight. Finally, Section 5 addresses the application and usefulness of our results and outlines ideas for future research.

2. System Model

2.1. Spatial Node Distribution

The spatial distribution of the nodes is given by a random point process on an infinitely large system plane. We use a homogeneous Poisson point process of density ρ nodes per unit area. This process is defined by the following two properties [8]:

- The number of nodes N in each finite subarea $\mathbf{A}$ of size $\|\mathbf{A}\| = A$ follows a Poisson distribution, i.e.,

$$P\,(n\,\text{nodes in}\,\mathbf{A}) \;=\; P\,(N=n) \;=\; \frac{\lambda^n}{n!}\,e^{-\lambda} \quad ;\, n \in \mathbb{N}_0\,, \tag{1}$$

  with an expected value $E\{N\} = \lambda = \rho A$.

- The number of nodes N_i in disjoint (non-overlapping) areas $\mathbf{A}_i$, $i \in \mathbb{N}$, are independent random variables, i.e.,

$$P\,(N_1 = n_1 \wedge N_2 = n_2 \wedge \ldots \wedge N_\kappa = n_\kappa) \;=\; \prod_{i=1}^{\kappa} P\,(N_i = n_i)\;. \tag{2}$$

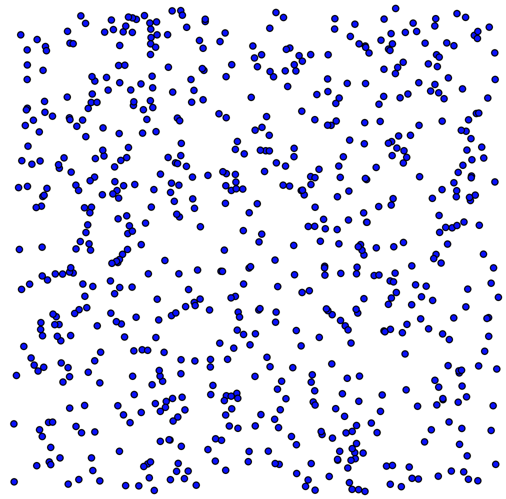


Figure 2. Illustration of Poisson distributed nodes

We denote this process as being homogeneous, if ρ is constant over the entire infinitely large area. In other words, the outcome of the random variable N only depends on the size of the subarea $\mathbf{A}$ but not on its particular location or shape. Figure 2 illustrates a square subarea of a homogeneous Poisson point process.

Recall that a homogeneous Poisson point process can be regarded as the limiting case of a uniform distribution of n nodes on an area of size A, as n and A tend to infinity but their ratio $\rho = n/A$ remains constant.

2.2. Wireless Channel Model

To describe the used channel model, we consider two nodes u and v that are located at a relative distance $s(u, v)$. Node u transmits a signal with power $p_t(u)$, and node v receives the signal with power $p_r(v)$. The signal attenuation between the nodes is defined as

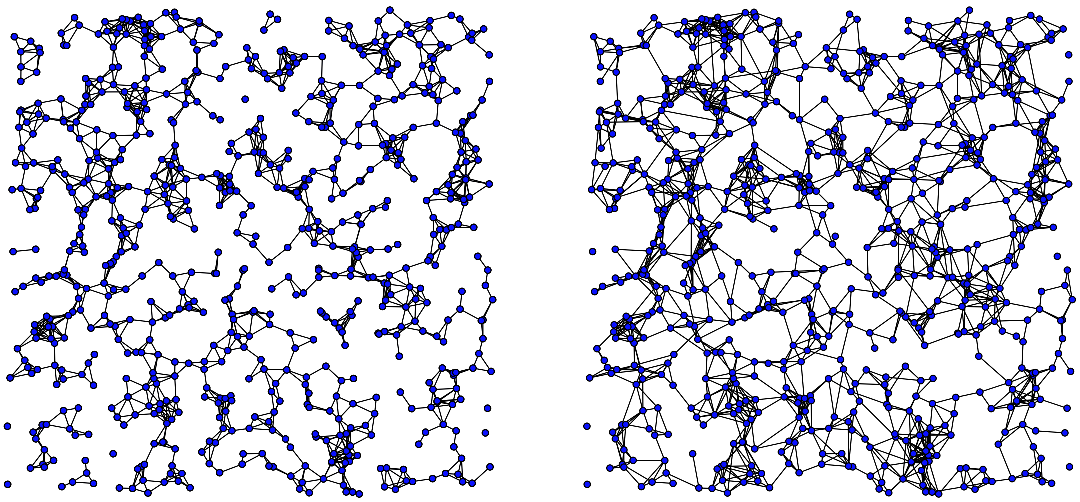
$$\beta(u, v) = \frac{p_t(u)}{p_r(v)} \, . \tag{3}$$

This can be expressed in terms of dB as

$$\beta(u, v) = 10 \log_{10} \left(\frac{p_t(u)}{p_r(v)} \right) \text{ dB} \, . \tag{4}$$

In a shadow fading environment, $\beta(u, v)$ comprises two components: a deterministic geometric component $\beta_1(u, v)$ and a stochastic component β_2. The geometric component is given by

$$\beta_1(u, v) = \alpha \; 10 \log_{10} \left(\frac{s(u, v)}{1 \text{ m}} \right) \text{ dB} \tag{5}$$



a. Purely geometric link model
($\beta_{th} = 69\,\mathrm{dB}$, $\alpha = 3$, $\sigma = 0$)

b. Shadow fading link model
($\beta_{th} = 69\,\mathrm{dB}$, $\alpha = 3$, $\sigma = 4$)

Figure 3. Network topology with $\rho = 4.375 \cdot 10^{-5}\,\mathrm{m}^{-2}$ on $4000 \times 4000\,\mathrm{m}^2$ with $r_0 = 200\,\mathrm{m}$

with pathloss exponent α. The stochastic component β_2 is chosen from a log-normal probability density function [13, 24]. Hence, β_2 in dB is chosen from a normal probability density function, i.e.,

$$f_{\beta_2}(\beta_2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\beta_2^2}{2\sigma^2}\right) . \tag{6}$$

Typical values for the standard deviation σ range up to 10 dB [13]. The total attenuation is

$$\beta(u,v) = \beta_1(u,v) + \beta_2 . \tag{7}$$

Node v receives the signal of node u properly, if $p_r(v)$ is larger than or equal to a certain threshold power $p_{r,th}(v)$, denoted as receiver sensitivity. If $p_r(v) \geq p_{r,th}(v)$, we say that node u establishes a wireless link to node v. In the following, we assume that channels are symmetric and all nodes have the same p_t and $p_{r,th}$. All links are thus considered as being undirected. Two nodes that have a link between each other are called neighbors in the network topology.

Figures 1 b and 3 b illustrate example topologies generated by the shadow fading model. Due to the random component, the following two events are now possible: there may be

- a link between two nodes that are more than r_0 away from each other;
- no link between two nodes that are located within distance r_0.

Using $\sigma = 0$, we obtain the purely geometric link model (see Fig. 3 a).

3. Link Probability and Node Degree

3.1. Link Probability Between Nodes

For given p_t and $p_{r,th}$, two nodes u and v are neighbors if the attenuation between them fulfills

$$\beta(u,v) \leq \beta_{th} \; , \tag{8}$$

with the threshold attenuation

$$\beta_{th} = 10 \log_{10} \frac{p_t}{p_{r,th}} \,\mathrm{dB} \, . \tag{9}$$

Let $\Lambda(u,v)$ denote the event that there is a link between u and v. If the Euclidean distance $s(u,v)$ between these two nodes is known, the probability for $\Lambda(u,v)$ is

$$P\left(\Lambda(u,v) \mid s(u,v)\right) = P\left(\beta(u,v) \leq \beta_{th} \mid s(u,v)\right) \tag{10}$$

$$= \int_{-\infty}^{\beta_{th}-\beta_1} f_{\beta_2}(\beta_2)\, d\beta_2 \tag{11}$$

$$= \int_{-\infty}^{\beta_{th}-\alpha 10 \log_{10}(s(u,v)/\mathrm{m})\,\mathrm{dB}} f_{\beta_2}(\beta_2)\, d\beta_2 \quad . \tag{12}$$

This yields

$$P\left(\Lambda(u,v) \mid s(u,v)\right) = \frac{1}{2} - \frac{1}{2} \,\mathrm{erf}\left(\frac{10\,\alpha}{\sqrt{2}\sigma} \log_{10} \frac{s(u,v)}{r_0} \,\mathrm{dB} \right) , \tag{13}$$

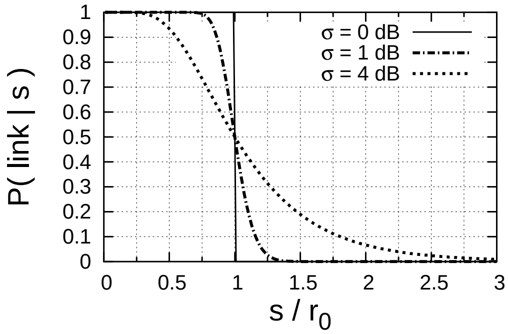
where the normalization term

$$r_0 = 10^{\frac{\beta_{th}}{\alpha\, 10\,\mathrm{dB}}}\,\mathrm{m} \tag{14}$$

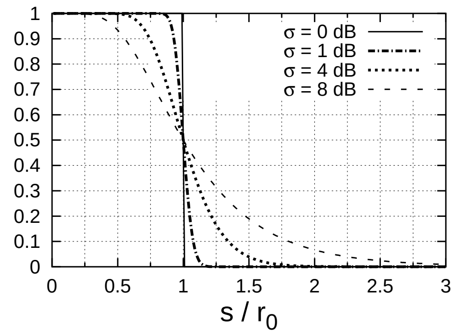
is the maximum distance granting a link in the absence of shadow fading ($\sigma = 0$), and the term $\mathrm{erf}(\cdot)$ denotes the error function, defined by

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp\left(-\zeta^2\right) \, d\zeta \, . \tag{15}$$

Figure 4 shows the link probability over $s(u,v)/r_0$ for $\alpha = 2$ and 4 with different values of σ. For $\alpha = 4$ and $\sigma = 8\,\mathrm{dB}$, there is still a link probability of more than 5 % at a distance $s = 2r_0$.



(a) Path loss exponent $\alpha = 2$



(b) Path loss exponent $\alpha = 4$

Figure 4. Probability that two nodes with given distance can establish a link

3.2. Number of Neighbors of a Node

The number of neighbors of a node is called its degree D; it is Poisson distributed according to (1). The expected value of D can be computed by integrating $\rho\, P\left(\Lambda(u,v) \mid s(u,v)\right)$ over the entire system plane [20], i.e.,

$$E\{D\} = \rho \int_0^{2\pi} \int_0^{\infty} P\left(\Lambda \mid s\right)\ s\, ds\, d\phi = 2\pi\rho \int_0^{\infty} P\left(\Lambda \mid s\right)\ s\, ds\,. \qquad (16)$$

We are especially interested in the probability that a randomly chosen node has no neighbor at all. This isolation probability is given by

$$P\left(\text{iso}\right) = P(D = 0) = e^{-E\{D\}}. \qquad (17)$$

Figure 5 shows some plots of $P\left(\text{iso}\right)$ over the node density ρ for some typical channel environments. These plots have been computed solving (16) by numerical means and plugging into (17). For example, in an environment with $\alpha = 3$, $\sigma = 4\,\text{dB}$, and $\beta_{th} = 50\,\text{dB}$, a density $\rho > 2.8 \cdot 10^{-4}\,\text{m}^{-2}$ is needed to limit the isolation probability to 10 %. If the channel has a path loss exponent $\alpha = 4$, the density must be increased to $\rho = 2.1 \cdot 10^{-3}\,\text{m}^{-2}$ to achieve the same $P\left(\text{iso}\right)$. If we keep $\alpha = 3$ but decrease the threshold attenuation to $\beta_{th} = 30\,\text{dB}$ — this corresponds, for instance, to a Bluetooth device with $p_t = 100$ mW and $p_{r,th} = 100\ \mu$W — a much higher density is required ($\rho = 6 \cdot 10^{-3}\,\text{m}^{-2}$). On the other hand, if $\sigma = 8\,\text{dB}$ instead of $4\,\text{dB}$, the isolation probability $P\left(\text{iso}\right)$ decreases because nodes can establish links to neighbors that are further away.

We also perform a number of computer-based simulations for $P\left(\text{iso}\right)$. The results, depicted as dots in Figure 5, validate the analytical results. The simulation methodology will be explained in Section 4.3.

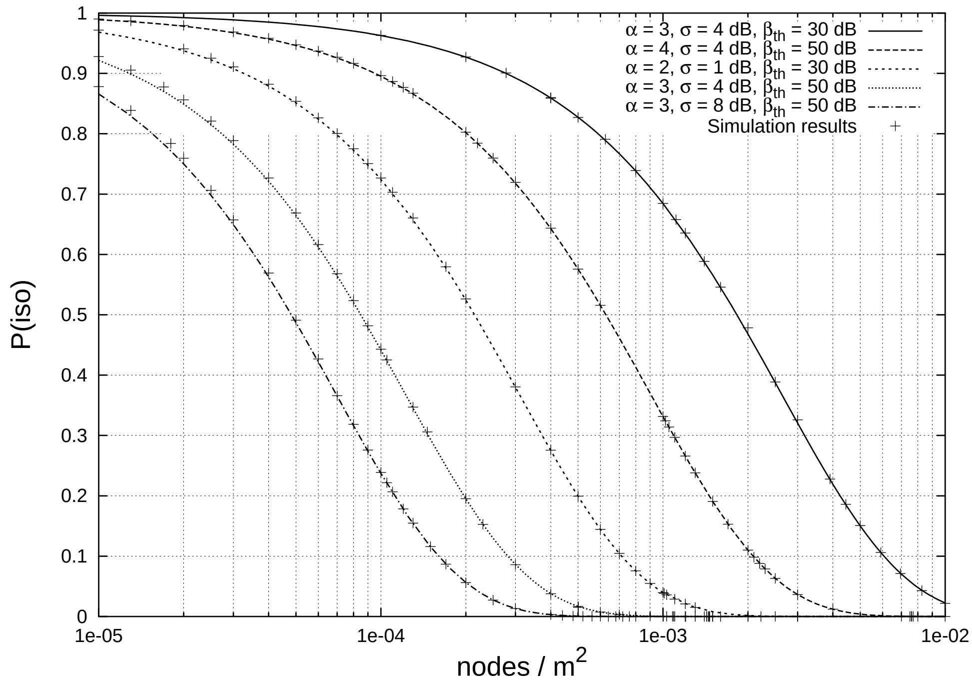


Figure 5. Probability that a randomly chosen node is isolated

4. Connectivity of a Subarea Network

4.1. Problem Statement

We now consider a subarea **A** of the infinite system plane, e.g., a circular area, as shown in Figure 6. The network formed by the nodes in **A** is said to be *connected* if and only if there is a path between each pair of nodes in **A**. Nodes outside of **A** can act as relay nodes to connect nodes inside of **A**.

For given α, σ, β_{th}, and A, we are interested in the minimum value of the node density ρ that yields, with high probability $P(\text{con}) = 99\,\%$ ("almost surely"), a connected subnetwork in **A**.

4.2. A Lower Bound for the Critical Node Density

The non-existence of isolated nodes is a necessary but not sufficient condition for a network to be connected. Thus, the probability that none of the nodes in **A** is isolated, denoted by $P\,(\text{no node iso})$, is an upper bound for the probability that all nodes in **A** are connected. That is, $P(\text{con}) \leq P\,(\text{no node iso})$. Consequently, the node density required to achieve, with a certain probability p, no isolated node is a lower bound for the node density required to achieve, with the same probability p, a connected network, i.e.,

$$\rho(P(\text{con}) = p) \;\geq\; \rho\,(P\,(\text{no node iso}) = p)\,. \tag{18}$$

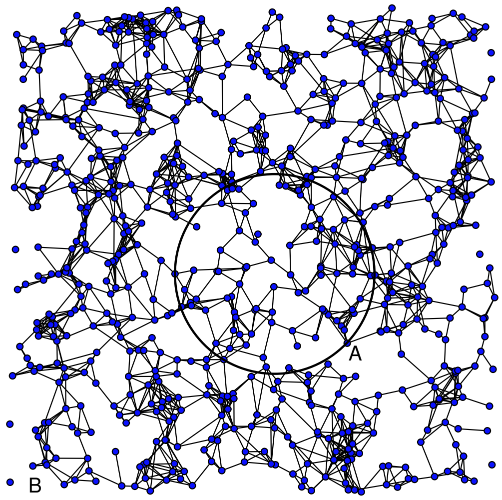


Figure 6. All nodes inside the subarea **A** are connected

In the following we calculate the right hand side for a probability p close to one.

The number of nodes in **A** is described by the random variable N, which follows a Poisson distribution (1) with an expected value $E\{N\} = \lambda = \rho A$. We require that many nodes are located in **A**, say $\lambda \geq 100$. To be able to achieve p close to one, the individual node isolation probability $P\,(\text{iso})$ must be very small. Given these assumptions, the isolation of different nodes can be considered to be almost independent events. Hence, we can state the conditional probability

$$P\,(\text{no node iso} \mid N = n) = (1 - P\,(\text{iso})\,)^n\,, \tag{19}$$

assuming that the number of nodes in **A** is known (i.e., $n = N$). To obtain the unconditional non-isolation probability, we write

$$P\,(\text{no node iso}) \;=\; \sum_{n=0}^{\infty} P\,(\text{no node iso} \mid N = n)\;P(N\!=\!n)\,, \tag{20}$$

which yields the double-exponential expression

$$P\,(\text{no node iso}) = \exp\Big(-\rho A\,P\,(\text{iso})\Big) = \exp\Big(-\rho A\,\exp\,(-2\pi\rho\,\xi)\Big) \tag{21}$$

with $\xi = \int_0^\infty P\,(\Lambda \mid s)\;s\,ds$.

Solving this equation for ρ leads to the result

$$\rho(P\,(\text{no node iso}) = p\,) = -\frac{1}{2\pi\xi}\,\mathrm{w}_{-1}\left(\frac{2\pi\xi\,\ln p}{A}\right)\,, \tag{22}$$

where $\mathrm{w}_{-1}(\cdot)$ denotes the real-valued, non-principal branch of the LambertW function[1], and $\ln(\cdot)$ is the natural logarithm. The fact that the LambertW function is implemented in several computer tools for symbolic mathematics eases the computation of $P\,$(no node iso) significantly.

In summary, this section has derived an explicit analytical lower bound for the critical node density that is required for almost sure connectivity. Let us now analyze how tight this bound is.

4.3. Tightness of Bound

For the case of no fading ($\sigma = 0$), we have shown in [2,3] that Penrose's mathematical theorems on the connectivity of random disk graphs [21] can be employed. It says the following: Regarding uniformly distributed nodes with fixed transmission range r_0 on a bounded area, we increase r_0 in each node synchronously (starting at $r_0 = 0$), such that links are added to the network in the order of increasing length. With probability close to 1, we obtain a connected network *at the same moment* when we obtain a network with no isolated nodes (if ρ is large). We can also interpret this theorem as follows: for fixed r_0, the node density ρ required to achieve $P\,$(no node iso) close to one is a very tight lower bound for the node density required to achieve $P(\mathrm{con})$ close to one.

An analogous phenomenon is known for pure (non-geometric) random graph processes, in which links between nodes are added uniformly at random (according to Erdös and Rényi [5,12]). Also in this case, the network gets connected, with high probability, at the moment when we add the link connecting the last isolated node.

Unfortunately, the fading channel model used in the article at hand creates a network graph that can neither be modeled by the purely geometric (Penrose) nor by the purely random (Erdös-Rényi) model. It is a "mixture" between both models for graph creation, since it has a geometric as well as a random component in order to add links. Let us therefore investigate the tightness of the bound by means of simulation.

In doing so, it is certainly not feasible to deploy an infinitely large simulation area. A method to simulating a *subarea* $\mathbf{A}$ of a homogeneous Poisson process, however, follows directly from its definition given in Section 2.1: As shown in Figure 6, we consider a simulation area $\mathbf{B}$ (of size B) that includes a circular observation area $\mathbf{A}$ in the middle. To avoid disturbing border effects and to ensure that sufficiently many relay nodes are located outside of $\mathbf{A}$, the distance from the border of $\mathbf{A}$ to the border of $\mathbf{B}$ is everywhere higher than the radius of $\mathbf{A}$. We choose a random number of nodes M from a Poisson distribution (1) with mean $E\{M\} = \rho B$. These M nodes are placed using a uniform random distribution over the entire area $\mathbf{B}$. The expected number of nodes placed in $\mathbf{A}$ is then $E\{N\} = \rho A$.

[1] The definition of the LambertW function is that it satisfies $\mathrm{w}(x)\, e^{\mathrm{w}(x)} = x$. If x is a real number, two real values for $\mathrm{w}(x)$ are possible for $-e^{-1} \le x \le 0$: the principal branch $\mathrm{w}_0(x)$ with $\mathrm{w}_0(x) \ge -1$, and a 2nd branch $\mathrm{w}_{-1}(x)$ with $\mathrm{w}_{-1}(x) \le -1$. In our problem, $\mathrm{w}_{-1}(x)$ yields the correct result.

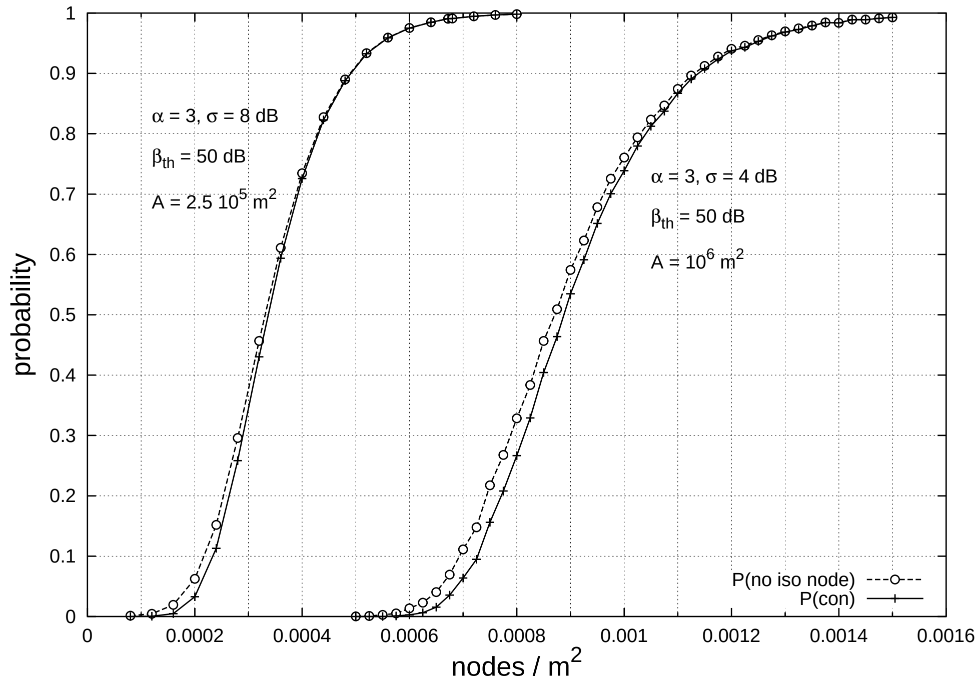


Figure 7. Comparison of $P(\text{con})$ and $P\,(\text{no node iso})$ on area of size A

To ensure $E\{N\} \geq 100$, as required above, we must set $E\{M\} \geq 100\frac{B}{A}$. Once the nodes are distributed, we establish all links according to the channel and transmission parameters subject to evaluation. We then check in this topology (a) whether there is an isolated node in **A**, and (b) whether all nodes in **A** are connected. We repeat the experiment $\Omega = 10\,000$ times. Finally, we compute the percentage of topologies with no isolated node and the percentage of connected topologies. These percentages serve as good estimates for $P\,(\text{no node iso})$ and $P(\text{con})$, respectively. We then increase the node density ρ and repeat the entire process.

The simulation results for two different scenarios are shown in Figure 7. There is a non-negligible difference between $P\,(\text{no node iso})$ and $P(\text{con})$ at low probability values, but both curves converge for higher probabilities. With respect to the critical node densities, this means that

$$\rho(P(\text{con}) = p) \;=\; \rho\,(P\,(\text{no node iso}) = p) \,+\, \epsilon\,, \quad \text{with } \epsilon \geq 0$$
$$\text{and } \epsilon \to 0 \;\text{ as }\; p \to 1\,. \tag{23}$$

Further simulation results of $\rho(P(\text{con}) = p)$ and $\rho(P\,(\text{no node iso}) = p)$ for $p = 99\,\%$ are reported in Table I. In all scenarios the difference ϵ is negligible. The comparison of simulation and theory also validates the correctness of the analytical expression for $\rho(P\,(\text{no node iso}) = p)$.

Table I. Critical Node Densities: Tightness of Bound and Comparison of Analytical and Simulation-Based Results

α	σ in dB	β_{th} in dB	A in m^2	$\rho\,(P(\mathrm{con}) = 99 \pm 0.05\,\%)$ in m^{-2} simulation	$\rho\,(P\,(\text{no node iso}) = 99 \pm 0.05\,\%)$ in m^{-2} simulation	with (22)
3	0	50	10^5	$1.42 \cdot 10^{-3}$	$1.39 \cdot 10^{-3}$	$1.41 \cdot 10^{-3}$
3	0	50	$2.5 \cdot 10^5$	$1.56 \cdot 10^{-3}$	$1.54 \cdot 10^{-3}$	$1.56 \cdot 10^{-3}$
3	4	50	10^5	$1.1 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$	$1.14 \cdot 10^{-3}$
3	4	50	10^6	$1.45 \cdot 10^{-3}$	$1.45 \cdot 10^{-3}$	$1.45 \cdot 10^{-3}$
3	8	30	$2.5 \cdot 10^5$	$1.97 \cdot 10^{-2}$	$1.97 \cdot 10^{-2}$	$1.96 \cdot 10^{-2}$
3	8	50	10^5	$6.15 \cdot 10^{-4}$	$6.15 \cdot 10^{-4}$	$6.05 \cdot 10^{-4}$
3	8	50	$2.5 \cdot 10^5$	$6.6 \cdot 10^{-4}$	$6.6 \cdot 10^{-4}$	$6.7 \cdot 10^{-4}$
4	4	50	10^4	$7.8 \cdot 10^{-3}$	$7.8 \cdot 10^{-3}$	$8.1 \cdot 10^{-3}$
4	4	50	10^5	$1.05 \cdot 10^{-2}$	$1.05 \cdot 10^{-2}$	$1.05 \cdot 10^{-2}$

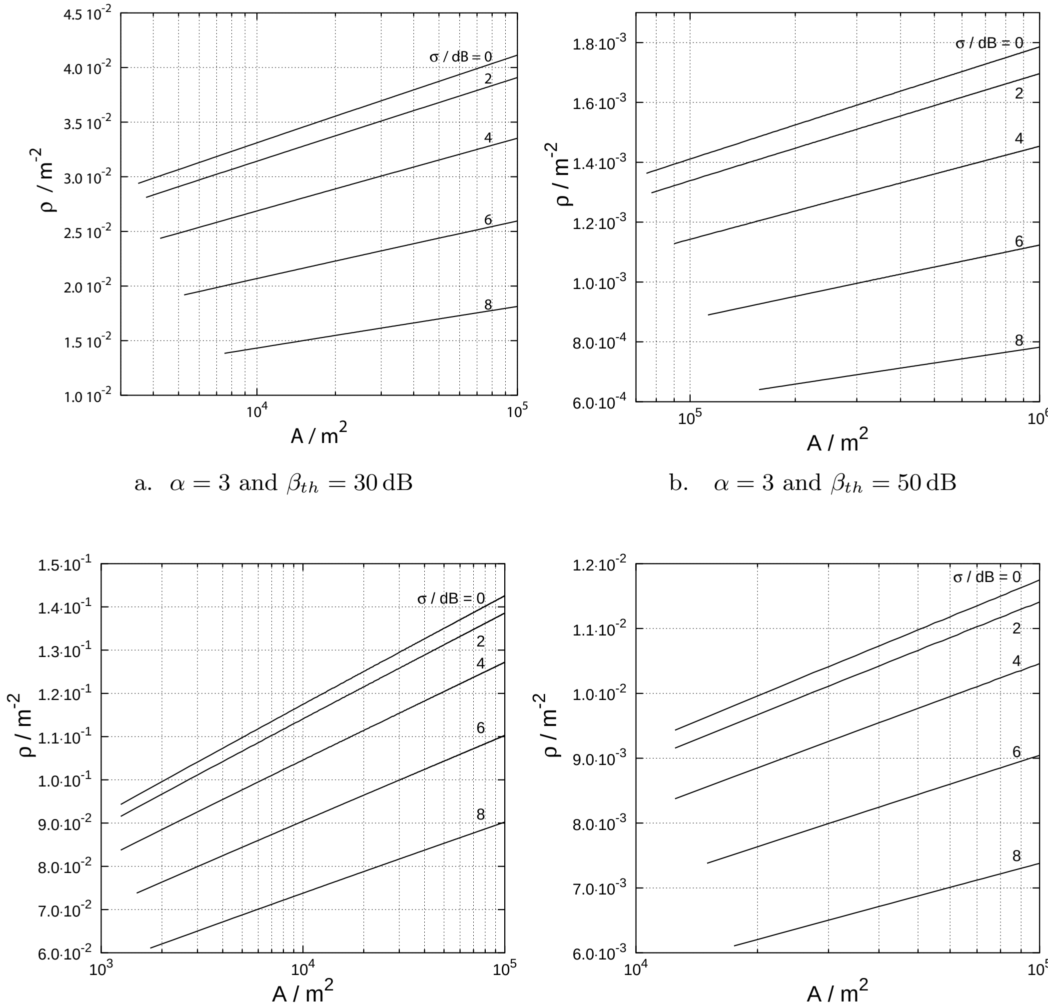
In conclusion, we can state that it is sufficient to compute the minimum density ρ required to achieve $P\,(\text{no node iso}) = 99\,\%$ and use this density as a tight lower bound for the density required to achieve $P(\mathrm{con}) = 99\,\%$.

4.4. Results and Interpretation

Given these observations, we employ (22) to compute $\rho(P\,(\text{no node iso}) = 99\,\%)$ over A for a number of typical channel environments. The final results are shown in Figures 8 a–d. Clearly, both the area size A and the fading variance σ have significant impact on connectivity. While the qualitative influence of A is straightforward, the influence of σ needs deeper investigation.

Let us analyze a scenario with $\beta_{th} = 50\,\mathrm{dB}$ in an environment with $\alpha = 3$ (Fig. 8 b). We would like to obtain an almost surely connected network that includes all nodes in an area of size $A = 10^5\,\mathrm{m}^2$. Without fading, a node density of $\rho = 0.0014\,\mathrm{m}^{-2}$ is required. If we increase σ, a *lower* node density is sufficient to make the network connected with the same high probability (e.g., $\rho = 0.0011\,\mathrm{m}^{-2}$ for $\sigma = 4$). In other words: a high fading variance "helps" the network to become connected.

This intuitively surprising insight can be explained as follows: Regarding a given node, fading takes away links to nodes that are located within a distance r_0 around a given node, but, in turn, it adds links to nodes that are located further away. On average, the number of added links is higher than the number of removed links, because the number of nodes located at a distance interval $[s, s+\delta]$ (with infinitely small δ) increases linearly with s. In summary, a higher σ increases the expected number of links of a node, it thus reduces the isolation probability,



a. $\alpha = 3$ and $\beta_{th} = 30\,\text{dB}$

b. $\alpha = 3$ and $\beta_{th} = 50\,\text{dB}$

c. $\alpha = 4$ and $\beta_{th} = 30\,\text{dB}$

d. $\alpha = 4$ and $\beta_{th} = 50\,\text{dB}$

Figure 8. Critical node density $\rho\,(P\,(\text{no node iso}) = 99\,\%) = \rho\,(P(\text{con}) = 99\,\%) - \epsilon$

which in turn improves the overall connectivity probability $P(\text{con})$. We must keep in mind, however, that this phenomenon only holds because our channel model decouples σ from α. In a real-world scenario, a higher σ typically comes with a higher pathloss α, which in turn dramatically reduces the connectivity.

A related insight has been reported in [6][2] with respect to a slightly different problem. That paper analyzes the existence of an infinitely large connected component using methods from continuum percolation. In that case, isolated nodes are allowed, but it is required that the whole system plane contains an unbounded

[2] The paper [6] appeared in parallel to the workshop version of this article [4].

connected subnetwork. The authors conclude that anisotropic radiation patterns allow such an infinite component to appear at a lower expected node degree than perfect circular coverage does. In that case, the non-zero occurrence probability of links that are longer than r_0 helps the network to percolate messages.

5. Summary and Further Work

This article studied the connectivity of multihop radio networks. As opposed to previous research in this field, we took into account stochastic shadowing effects between the nodes. Using a combination of analytical and simulation-based methods, we gave insight about the impact of shadowing on connectivity: for a given pathloss exponent α, a higher fading variance σ improves the connectivity behavior, i.e., a lower node density is sufficient to achieve a connected network.

The computed values of the critical node density are of practical relevance for the design and simulation of wireless multihop networks. For example, the results can be applied for the "planning" of sensor dust networks, which may consist of thousands of sensors that are randomly distributed over a particular area that needs to be monitored. For a given sensor technology (having a certain p_t and $p_{r,th}$) and a given channel environment, we now can determine the minimum density ρ that is needed to achieve a connected sensor network covering a certain area. Even if a fully connected network is not desired, the above results give us a notion of the "level of connectivity" in the network.

The critical node densities are also valid for mobile nodes, if the node mobility retains the spatial node distribution (see [1] for a discussion on this issue). In this case, $P(\text{con})$ gives us the percentage of time that the network is connected, i.e., $P(\text{con}) = 99\,\%$ means that the network is connected during almost the complete time. Such a setup is sometimes desired in simulation-based performance evaluation of routing protocols [16–18]. Last but not least, the results are also useful if power or topology control [23] is employed. We then interpret p_t as the *maximum* possible p_t of a certain node type.

Several issues remain for future work in this area. First, it is interesting to relax the requirement that *all* nodes are connected. In a scenario where some nodes have special functions (e.g., access to the fixed network), we are satisfied if each node has a path to at least one of these dedicated nodes. Second, the results of this article and the impact of interference on connectivity [10] could be combined to a common framework. Last but not least, additional understanding of the connectivity dynamics, e.g., the stochastic properties of the duration of a path between two nodes [14, 19, 25], needs further investigation.

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Authors' Vitae

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