

Dynamic Model and Motion Control Analysis of Three-link Gymnastic Robot on Horizontal Bar

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Abstract

Based on the observation of gymnast's swing-up movement, a simplified three-link horizontal bar robot is designed. With corresponding mathematic model developed, its dynamic characteristics are analyzed and simulation results are presented. From a general control perspective, an effective approach, that combines Human simulated intelligent control method with Schema theory, and its feasibility are pointed out to obtain the planning motion control.

1 Introduction

Through training and repeated practice, a gymnast can complete some complicated and skillful motions on gymnastic appliances such as body's curling, giant swing, swing-up and handstand, etc. Meanwhile, magnitude and direction of forces generated by body joints are harmonized and controlled with very high skills and sensory-motor intelligence. Therefore, study and understand their body movement and related joint's action are very helpful to develop the intelligent control theory for autonomous robots motion control.

As a major mechanism for studying such kind of motion that simulating human on the gymnastic apparatus, gymnastic robots, including rings [1] and horizontal bar gymnastic robots, have been proposed for many years, and some significant results can be found. Acrobot, a two-degree-of-freedom planar robot with a single actuator at its coxa, but no actuator at the wrist is discussed in [2~5], in which fuzzy control, heuristic control, partial feedback linearization and control method based on energy are applied respectively to finish planning trajectories of swing-up and inverted equilibrium.

According to the analysis of gymnast's skills and sensory-motor intelligence, however, the gymnastic robot model with only two joints is not enough to demonstrate human movement on the horizontal bar because it is apparent that athlete's shoulder joints play a very important role in controlling trunk's action. Briefly, in the process of body swing-up and being inverted, the shoulder can bend so that certain angle is formed between arms and trunk, and with

torque cooperation from the coxa, this predetermined motion can be finished accurately. Certainly, on the point of bionics, human joints have no ability to drive a free 360-degree rotation, which are under the limits of strength and angle. Therefore, all these limiting factors should be represented in the gymnastic robot for obtaining more accurate physical model.

In this paper, horizontal bar gymnastic robot, with three links, including two active joints and a passive joint, is presented, and its mathematic model is derived in details, further, two kinds of typical motion behaviors, free move and forced move, are simulated in the section II. Considering conditions of torque strength and angular limits, in section III, we demonstrate a set of complete trajectory parameters analysis based on the gymnast swing-up and handstand on the horizontal bar. Finally, a control method with integration of Human-Simulated Intelligent Control method and Schema Theory of Cognitive Science is proposed.

2 Three-link gymnastic robot dynamics

Fig. 1 shows the two-dimensional coordinates map for the horizontal bar gymnastic robot, in which the model consists of three links and three joints. Shoulder and coxa are active limited joints with rotation range of $-5\sim 225$ degrees for the angle between arm and trunk, and $-5\sim 180$ degrees for the angle between leg and trunk; the pivot connecting hand and bar is passive joint, which has $0\sim 360$ degrees rotation range.

Assuming L_i , l_i , m_i , J_i and θ_i are respectively link length, distance from the centroid to corresponding axis, mass, moment of inertia computed from three link centroid, and angle deviated from vertical direction clockwise, where, notation $i=1,2,3$ represents three links respectively. Besides, the motors mounted on the two active joints have masses mc_1 and mc_2 respectively, and the frictional torque coefficients of three joints are notated as c_1 , c_2 and c_3 . u_1 and u_2 are symbolized as torques from two active joints which clockwise rotation is assumed to be positive.

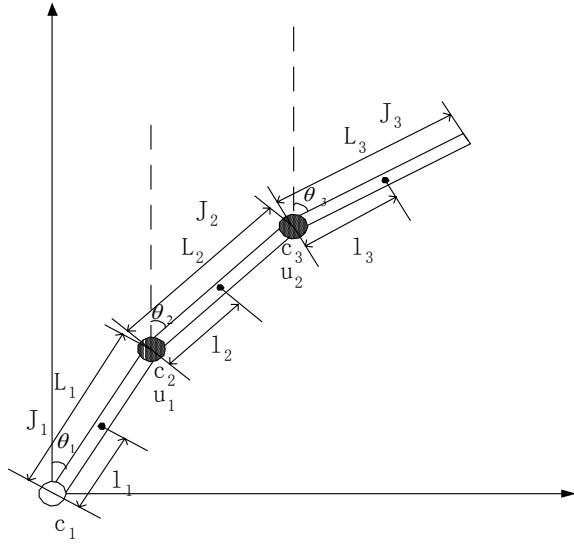


Fig.1 Coordinates map for three-link horizontal bar gymnastic robot

Then, the Lagrange equation for this model can be expressed as

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{X}_i} \right) - \frac{\partial T}{\partial X_i} + \frac{\partial V}{\partial X_i} + \frac{\partial D}{\partial \dot{X}_i} = U_i \quad i=1,2,3 \quad (1)$$

where,

$$X = [\theta_1 \quad \theta_2 \quad \theta_3]^T, U = [-u_1 \quad u_1 - u_2 \quad u_2]^T;$$

The system kinetic energy T under the generalized coordinates, the potential function V and the energy dissipated D by joint frictions can be obtained through the equations (2), (3) and (4) respectively:

$$T = \frac{1}{2} \sum_{i=1}^3 \left[J_i \dot{\theta}_i^2 + m_i \left[\left(\frac{d}{dt} \left(\sum_{k=i-2}^{i-1} L_k \sin \theta_k + l_i \sin \theta_i \right) \right)^2 + \left(\frac{d}{dt} \left(\sum_{k=i-2}^{i-1} L_k \cos \theta_k + l_i \cos \theta_i \right) \right)^2 \right] + \frac{1}{2} \sum_{i=1}^2 m c_i \left[\left(\frac{d}{dt} \sum_{k=i-1}^i L_k \cos \theta_k \right)^2 + \left(\frac{d}{dt} \sum_{k=i-1}^i L_k \sin \theta_k \right)^2 \right] \right] \quad (2)$$

$$V = \sum_{i=1}^3 m_i g \left(l_i \cos \theta_i + \sum_{k=i-2}^{i-1} L_k \cos \theta_k \right) + \sum_{i=1}^2 m c_i g \left(\sum_{k=i-1}^i L_k \cos \theta_k \right) \quad (3)$$

$$D = \frac{1}{2} \sum_{i=1}^3 c_i (\theta_i - \theta_{i-1})^2 \quad (4)$$

By plugging these parameters into the equation (1), we obtain the following nonlinear system dynamic equation:

$$A(X) \ddot{X} = B(X, \dot{X}) \dot{X} + C(X) + D \quad (5)$$

where,

$$A(X) = \begin{bmatrix} A_{11} & A_{12} \cos(\theta_2 - \theta_1) & A_{13} \cos(\theta_3 - \theta_1) \\ A_{12} \cos(\theta_2 - \theta_1) & A_{22} & A_{23} \cos(\theta_3 - \theta_2) \\ A_{13} \cos(\theta_3 - \theta_1) & A_{23} \cos(\theta_3 - \theta_2) & A_{33} \end{bmatrix}$$

$$C(X) = \begin{bmatrix} C_1 \sin \theta_1 \\ C_2 \sin \theta_2 \\ C_3 \sin \theta_3 \end{bmatrix}$$

$$B(X, \dot{X}) = \begin{bmatrix} B_{11} & B_{12} \sin(\theta_2 - \theta_1) \dot{\theta}_2 & B_{13} \sin(\theta_3 - \theta_1) \dot{\theta}_3 \\ B_{21} \sin(\theta_2 - \theta_1) \dot{\theta}_1 & B_{22} & B_{23} \sin(\theta_3 - \theta_2) \dot{\theta}_3 \\ -B_{13} \sin(\theta_3 - \theta_1) \dot{\theta}_1 & B_{32} \sin(\theta_3 - \theta_2) \dot{\theta}_2 & B_{33} \end{bmatrix}$$

$$D = \begin{bmatrix} -u_1 \\ u_1 - u_2 \\ u_2 \end{bmatrix}$$

and A_i , B_i , C_i are constants related to the model physical parameters:

$$A_{11} = m_1 l_1^2 + J_1 + (m_2 + m_3 + m c_1 + m c_2) L_1^2$$

$$A_{12} = (m_2 l_2 + (m_3 + m c_2) L_2) L_1$$

$$A_{13} = m_3 l_3 L_1$$

$$A_{22} = m_2 l_2^2 + J_2 + (m_3 + m c_2) L_2^2$$

$$A_{23} = m_3 l_3 L_2$$

$$A_{33} = m_3 l_3^2 + J_3$$

$$B_{11} = -(c_1 + c_2)$$

$$B_{12} = c_2 + (m_2 l_2 + (m_3 + m c_2) L_2) L_1$$

$$B_{13} = m_3 l_3 L_1$$

$$B_{21} = c_2 - (m_2 l_2 + (m_3 + m c_2) L_2) L_1$$

$$B_{22} = -(c_2 + c_3)$$

$$B_{23} = c_3 + m_3 l_3 L_2$$

$$B_{32} = c_3 - m_3 l_3 L_2$$

$$B_{33} = -c_3$$

$$C_1 = (m_1 l_1 + (m_2 + m_3 + m c_1 + m c_2) L_1) g$$

$$C_2 = (m_2 l_2 + (m_3 + m c_2) L_2) g$$

$$C_3 = m_3 l_3 g$$

Apparently, three-link horizontal bar gymnastic robot is a typical multiple inputs (u_1 , u_2) & multiple outputs (θ_1 , θ_2 , θ_3) nonlinear system. From the equation (5), it is observed that there exists a very strong coupling among the variables (θ_1 , θ_2 , θ_3) in this system, and this coupling effect varies with link's motion.

Table 1 shows all physical parameters used in simulation based on software tool Simulink® in Matlab®.

Variable	Value	Unit	Variable	Value	Unit
L_1	0.25	m	mc_1	0.3	kg
L_2	0.25	m	mc_2	0.3	kg
L_3	0.5	m	J_1	0.0097395	kgm^2
l_1	0.125	m	J_2	0.011458	kgm^2
l_2	0.125	m	J_3	0.045833	kgm^2
l_3	0.25	m	c_1	0.7056	Nms
m_1	1.87	kg	c_2	0.7056	Nms
m_2	2.2	kg	c_3	0.7056	Nms
m_3	2.2	kg			

Table 1 Model physical parameters of three-link horizontal bar gymnastic robot

To observe the free motion behavior of this nonlinear system, we initialize three angles and no torque is executed. As shown in Figure 2 (a)~ (c), due to the effects of gravity and frictions, each of three links is stabilized gradually at the position pointing to the ground ($\theta_1 = \theta_2 = \theta_3 = \pi$) after damping oscillations. But, they do not swing down independently like a single pendulum because of the strong coupling among the joints. For example, when initial values for three angles are $\theta_1 = 110^\circ$, $\theta_2 = \theta_3 = 170^\circ$ respectively, θ_2 , θ_3 should increase to π firstly then continue increasing if only their own gravity and joint friction applied. However, because the gravity of first link (arm) generates much stronger torque

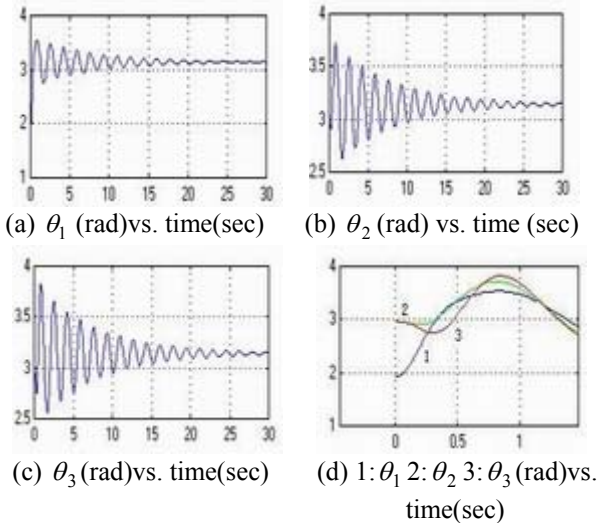


Fig.2 Free movement of horizontal bar gymnastic robot

than others resulting from this set of initial values, it obtains the bigger angular acceleration initially so that the 2nd and 3rd links (trunk and legs) have reverse acceleration.

Their trajectories are shown in Fig. 2 (d), in which unlike the first link, the 2nd and 3rd links move backward. The values of θ_2 and θ_3 are decreased at first then increased.

Furthermore, with a torque impulse executed on active joint, the robot has an initial velocity; still, each of three links is stabilized gradually at the position pointing to the ground after damping oscillations. Fig. 3 (a)~(c) show the varying process of the three angles while the torque drive $u_2=1$ occurs from $t=0.1s$ to $1.1s$. By the coupling between the joints, u_2 not only lets the 3rd link obtain an initial velocity, but also affects the first two links. But this positive (clockwise) torque u_2 effects on the 2nd link (trunk) negatively, and due to the u_2 is greater than the gravity torque of the 2nd link, θ_2 decreases during the period of execution of u_2 , until u_2 stops. In contrast, the 1st link can obtain a positive torque from the anticlockwise rotation of the 2nd link acting on the 2nd joint (shoulder). But this torque is small so that after a certain point the angle decreases because of gravity. Until u_2 vanishes and the 2nd link begins to rotate clockwise, the angle of the 1st link increases (Figure 3 (d)).

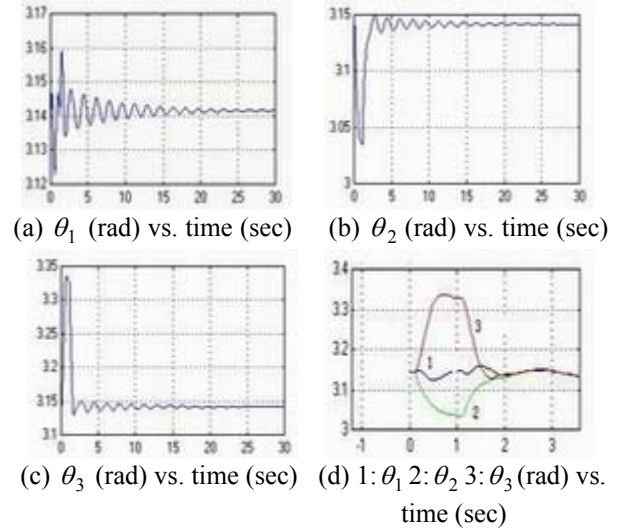


Fig.3 Forced movement of horizontal bar gymnastic robot

3 Planning Motion

In this paper, the concentrate planning motion for the control-limited three-link horizontal bar gymnastic robot is swing-up and inverted equilibrium, which can be described by means of characteristic states or energy transfer process of the system. To illustrate the system performance in this planning trajectory, we get the system state variable $s = (\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)^T$ and energy $E = [E_k \quad E_{p1} \quad E_{p2} \quad E_{p3}]$, so, the motion control requirement can be defined as:

$$x_0 = (\pi, \pi, \pi, 0, 0, 0) \xrightarrow{\text{control}} x_T = (0, 0, 0, 0, 0, 0)$$

or

$$E_0 = [0 \ 0 \ 0 \ 0] \xrightarrow{\text{control work contribution}}$$

$$E_T = [0 \ 2g(m_1l_1 + m_1l_1) \ 2g(m_2(l_2 + L) + m_2(L_1 + L_2)) \ 2gm_3(l_3 + L_1 + L_2)]$$

Where, E_k is the system total kinetic energy (rotation energy added), and E_{p1} , E_{p2} and E_{p3} are the potential energy of each link (including motors in joints) respectively.

The swing-up and inverted equilibrium control of horizontal bar gymnastic robot, studied in this paper, makes the gymnastic robot swing up from the stable equilibrium to the unstable fix point of this dynamic system, which does simulate the gymnast's motions. The control approach focuses on final accurate orientation for system states rather than specific values of the state and energy variables in the whole motion process. We can promote a high efficient method to reach the object position, if we study the moving and control skills used by the gymnast in finishing the same target.

Figure 4 (a ~ l) refers to a typical trajectory scheme of swing-up and inverted equilibrium motion usually performed by a gymnast in the game.

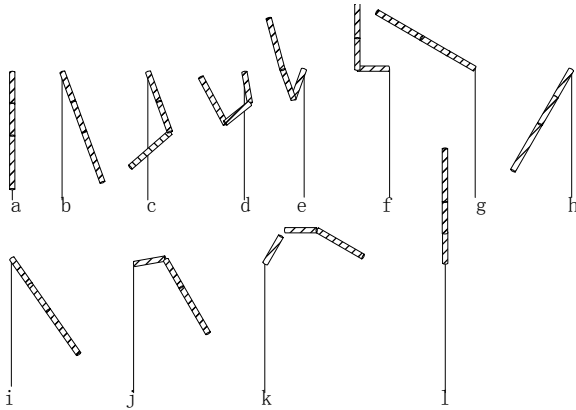


Fig.4 trajectory scheme of swing-up and inverted equilibrium motion

Beginning with swinging a small angle backward from the hang position ($\theta_1 = \theta_2 = \theta_3 = \pi$), through driving legs or legs and the trunk ($\theta_1 = \theta_2 = \theta_3 = \pi - \Delta\theta_1$), the gymnast obtains certain potential energy. In this step, initial negative torque is generated from shoulders and coxa joints, but it is unnecessary to keep the three parts in line.

And then, the gymnast raises the legs forward dramatically ($\theta_1 = \theta_2 = \pi - \Delta\theta_1, \theta_3 = \pi + \Delta\theta_2$), driven by the coxa. Further, the shoulders execute the torque to draw up the trunk following the legs so that the legs and the trunk are almost upright. At the same time the arms also lift, driven by the other two parts ($\theta_1 = 3\pi/2, \theta_2 = \theta_3 = 2\pi - \Delta\theta_3$).

Due to the torque strength limited, in general, the first swing cannot reach the inverted position. But the higher potential energy is obtained, which can be transferred to be kinetic energy. Then, by the torques from shoulders and coxa, the gymnast opens the body to a line for the purpose of swinging downward (shown in Fig. 4 (c ~g)). The positive torque from the coxa should be larger than that shoulders provide, which leads the leg's angular velocity greater than trunk's ($\omega_3 > \omega_2$) and the angle between them decreases (see Fig.4 (c ~d)). However, the leg angular velocity ω_3 must become less ($\omega_3 < \omega_2$) for adjusting the three moving parts in straight ($\theta_1 = \theta_2 = \theta_3 = 3\pi/2 + \Delta\theta_4$) (see Fig. 4 (d ~e)).

With assistance of gravity, the gymnast swings down and continues rotating anticlockwise after passing the lowest point to obtain certain velocity ($\theta_1 = \theta_2 = \theta_3 = \pi - \Delta\theta_4$).

Afterwards, the reverse torque of u_1 from shoulders causes the arms to be raised higher, and the angle θ_2 formed by arms and the trunk would increase firstly then decrease because of the difference between the torque strength from shoulder and coxa. Similar transition would occur at the angle θ_3 between the leg and trunk. Both two angles θ_2, θ_3 vary because the legs and trunk rotate faster than arms do, and finally, the whole body is stabilized at the inverted equilibrium position in straight (Figure 4 (g) to (l)).

So far, we know that the relation between the strength and direction of driving torques is highly dependent on the three angles and their speeds, which means that control strategy must be switched during the motion shifts.

4 Indeterminable of Motion Control and Description of Control Strategy

Horizontal bar gymnastic robot is a typical underactuated mechanical nonlinear system with fewer actuators than degrees of freedom. From a mathematic perspective, it consists of higher state dimensions than control input dimensions in generalized coordinates. In general, this kind of systems usually exhibits some features like complex internal dynamics, nonholonomic behavior, and lack of feedback linearization. Particularly, in our case, swing-up and inverted equilibrium of three-link gymnastic robot is not a simple transformation of several variables but a series behaviors and their connection. It involves cooperation among multiple control inputs and controlled variables. It requires the effective real time response and large numbers of real-time data processed. Hence, horizontal bar robot is a typical complex system.

Compared with car-inverted pendulum, another frequently studied nonlinear dynamic system, both of them have large-range unstable motion and their energies are accumulated by dramatic vibration. There are strong coupling in both systems, which help to implement control as well as to make the system unmanageable. However, there is an apparent difference between them.

The only drive in the car-pendulum system is the horizontal driving force generated from the car. The motion control of pendulum is completed by the weight of pendulum and the coupling between pendulum and car. Therefore, two control areas can be divided clearly by the position of pendulum being up or below the horizontal line, where, direction of pendulum acceleration, driven by the car moving, is opposite.

Unlike horizontal car moving, the driving torque for horizontal bar gymnastic robot comes from two active joints, which results in a more complicated control strategy requested. From the analysis in section III, we know that both angles at two active joints among these three links have to be concerned when we analyze torque direction, strength, and angular acceleration of each link. So, it is infeasible to establish sectional control just depending on angles. Gymnastic robot finishes swing-up and inverted equilibrium on horizontal bar through a series movement, in which one action or sequential actions is for reaching a step target, such as increasing or decreasing kinetic or potential energy, and angular velocity or acceleration to a certain value. We can specify series of motions as several sections, and each section has a predetermined control object, strategy and interfaces between them. Therefore, swing-up and inverted equilibrium motion control is a parallel and series combined complicated process for horizontal bar gymnastic robot, where control relation between each section is in series, but control of motions inside each section are in both series and parallel.

Schema theory derived from the cognitive science provides an effective method to process large-scale systemic complex. Through the works by Authors in [6] [7], in which they build this method with fuzzy control or neural network to control mobile robot or autonomous robot, it proves that an approach with integration of intelligent control method and schema theory is feasible to improve control performance for complex dynamic system. Human-Simulated Intelligent Controller has been applied to the pendulum under limited torque, single and two-linked car inverted pendulum successfully [8~10], which has big advantages on simplicity and applicability for solving problems in strong coupling nonlinear system. Therefore, after simulating sensory-motor intelligence exhibited in human brain and establishing serial motion transition schema, we expect to finish planning motion control for the three-link horizontal bar gymnastic robot by a new control strategy based on Human-Simulated

Intelligent Controller and Schema Theory. This work is in the progress and will be presented in the future article.

Conclusion:

In conclusion, a three-link horizontal bar gymnastic robot has been designed and its mathematic model developed. Simulation shows the feasibility of this model. Also, a control strategy is proposed based on analyzing gymnast's motion control on horizontal bar.

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