Volumetric Error Analysis and Architecture Optimization for Accuracy of HexaSlideType Parallel Manipulators

Jeha Ryu* and Jongeun Cha

Jeha Ryu* is with Department of Mechatronics, Kwangju Institute of Science and Technology (K-JIST), 1 Oryong-dong, Puk-gu, Kwangju 500-712, Republic of Korea (telephone: +82-62-970-2389, e-mail: ryu@kjist.ac.kr).

Jongeun Cha is with Department of Mechatronics, K-JIST(e-mail: gaecha@geguri.kjist.ac.kr).

Abstract—This paper presents a volumetric error analysis and an architecture optimization method for accuracy of parallel manipulators. A comprehensive volumetric kinematic error model that relates all kinematic error sources in the manipulator's architecture to the pose errors of the end-effector is derived for HexaSlide (*PUS*) type parallel manipulators. The error model results in the total error transformation matrix from which three Error Amplification Factors (*EAF*) are derived and used as design criteria for accuracy in the optimum design formulation with constraints on workspace and design variable limits. Then, design optimization for accuracy has been performed by using a nonlinear optimization technique. Optimization results have been validated by Monte Carlo statistical simulation technique.

Key Words: Parallel Manipulators, Accuracy, Error Analysis, Monte Carlo Simulation, Optimum Design.

I. INTRODUCTION

Optimal kinematic design of robot manipulators is very important for achieving desired kinematic performances.

As parallel manipulators have advantages of higher stiffness, better accuracy, etc. over serial manipulators, optimal architecture design of parallel manipulators, therefore, has drawn much attention in recent years. In the optimal design, maximization of the static kinematic performance measures such as dexterity, stiffness, workspace, isotropy, accuracy, working speed, and payload has been studied. Among them, dexterity has been considered important because it is a measure of a manipulator's ability to arbitrarily change its position and orientation or to apply forces and torques in arbitrary direction. Many researchers[1-6], therefore, have performed design

optimization focusing on the dexterity of parallel manipulators by minimization of the condition number of the Jacobian matrix. Workspace has also been considered as an important design criterion in designing some parallel manipulators[7-11] because they usually have smaller workspace compared to serial manipulators. Other researchers[12-15] considered dexterity, workspace, and stiffness simultaneously in the design of parallel manipulators. For example, Hong and Kim[15] used a performance measure combining the condition number with the manipulability ellipsoid volume in designing a rapid prototyping machine called EclipseTM.

Accuracy of a parallel manipulator is also an important design criterion for very fine motion tasks such as coordinate measurement, medical operation, machining operation, and micro-manipulation. Accuracy of a manipulator can be affected by many factors such as actuator control errors, installation errors, manufacturing tolerances and clearances, architecture design, etc. In order to investigate effects of manufacturing and actuation errors on the platform pose accuracy, Wang and Masory[16] presented a kinematic error model for a Stewart platform using the D-H convention. Ropponen and Arai[17] presented, for the modified Stewart platform, the closed kinematic error model that includes the joint position and actuation errors by differentiating the inverse kinematic equation. They then computed the error ellipsoids and the position errors of the end-effector. Patel and Ehmann[18] derived a more complete kinematic error model for the Stewart platform machine tool that accounts for all relevant error sources such as manufacture error, thermal and elastic deformation error, control error, etc. They then plotted error gain sensitivity as a design tool for tolerance allocation during manufacture. Bi et al. [19] presented an accuracy analysis for a serial-parallel micromotion manipulator and derived error effects that are useful for the design and manufacture of the manipulator. Kim and Choi[20] presented forward and inverse error bound analyses as well as associated eigenvalue problems for the Stewart platform. Xi and Mechefske[21] derived an error model for hexapods with fixed leg-lengths only focusing on the effect of the actuation errors and evaluated the platform pose errors by applying several matrix norms. They observed that the error transmission between the moving axes and the moving platform is closely related to the condition number of the Jacobian matrix. Pasek[22] analyzed the manipulator's accuracy based on the Monte Carlo statistical experiment approach and a Jacobian approximation. The aforementioned research about the accuracy, however, is mainly for evaluating the end-effector pose errors based on some kinematic error models.

While there have been many researches on the design optimization for dexterity, isotropy, load carrying capability, working speed, and workspace, there are, however, only a few researches on design optimization for accuracy. For

achieving good accuracy, we present an architecture design optimization method for parallel manipulators in this paper. Our approach is based on a comprehensive volumetric error model that includes all kinematic error sources such as actuation, installation, and manufacturing errors. The error model is derived from a differential inverse kinematic equation and a total error transformation matrix rather than the Jacobian matrix is derived for relating all error sources to the end-effector pose error. A detailed geometric interpretation is given to each error source that includes the actuator axis straightness error. Then, as the main performance criterion, global error amplification factors (*EAF*) that are derived from the singular value decomposition of the total error transformation matrix are used as design criteria for accuracy in the design optimization. The optimization problem is then formulated considering constraints on workspace and design variable limits. We applied the proposed design optimization technique, as an example, to a HexaSlide (*PUS*) type hexapod manipulator that has six moving sliders on the ground and six fixed leg-lengths connected to a platform. The Monte Carlo simulation evaluates the accuracy improvement by the proposed design optimization method.

This paper is organized as follows; in section II, the geometry and nomenclature of the HexaSlide manipulator are described. Section III derives a comprehensive volumetric model that includes all error sources. From the error model, a total error transformation matrix is defined for optimal design formulation. In section IV, accuracy design criteria in terms of error amplification factors are defined from the singular value decomposition of the total error transformation matrix. In section V, an optimization problem is formulated for the accuracy design. In section VI, the results of the optimization and validation by Monte Carlo simulation are presented. In section VII, conclusions are given.

II. DESCRIPTION OF THE HSM

A HexaSlide manipulator (HSM) shown in Fig.1 is composed of six constant-length links, a mobile platform, and six prismatic actuators fixed to the base. The links are connected to the actuators and to the platform by universal and spherical joints pair. This manipulator controls the pose of the platform by moving the spherical joints A_i , from $A_{i,0}$ to $A_{i,1}$ with six actuators. The base coordinate frame XYZ is placed on the center O of the base. The origin of the platform coordinate frame xyz is attached to the center C of the moving platform. Note that the point C may be a tip of machining tool or a tip of the touch probe in a coordinate measuring machine. All vectors and matrices will be denoted in bold letters. The pose of the platform can be expressed by a position vector $\mathbf{C}$ and a rotation matrix $\mathbf{R}$ as

$$\mathbf{C} = [X_C \ Y_C \ Z_C] \quad \mathbf{R} = \mathbf{R}_{X,\theta} \mathbf{R}_{Y,\psi} \mathbf{R}_{z,\phi} \quad (1)$$

where θ , ψ , and ϕ are three Euler angles that are chosen with respect to the global X , Y , and the local z axes, respectively.

III. VOLUMETRIC ERROR ANALYSIS

Kinematic errors in the manipulator architecture, such as manufacturing tolerances and clearances, joint installation errors, and actuator control errors, can lead to the pose errors of the mobile platform. In this section, a geometric relationship between the kinematic errors and the pose errors is derived.

Fig. 2 shows the i -th chain and kinematic parameters of the HSM. In this figure, vectors $\mathbf{a}_i$ and $\mathbf{n}_i$ are the unit vectors along the actuation and constant-length link directions, respectively, and λ_i is the articular variable that is the distance from $A_{i,0}$ to A_i points. A closed-loop vector equation of the i -th chain is then given by

$$\lambda_i \mathbf{a}_i = \mathbf{C} - \mathbf{R} \mathbf{B}'_i - \mathbf{A}_{i,0} - l_i \mathbf{n}_i \quad (2)$$

where $\mathbf{B}'_i$ is the position vector from B_i to C and is represented in the platform coordinate frame.

A differential error model is obtained by differentiating Eq.(2) as

$$\delta \lambda_i \mathbf{a}_i + \lambda_i \delta \mathbf{a}_i = \delta \mathbf{C} - \delta \mathbf{R} \mathbf{B}'_i - \mathbf{R} \delta \mathbf{B}'_i - \delta \mathbf{A}_{i,0} - l_i \delta \mathbf{n}_i - \delta l_i \mathbf{n}_i \quad (3)$$

where the derivative of the rotation matrix is given by

$$\delta \mathbf{R} = \delta \boldsymbol{\theta} \times \mathbf{R} = \begin{bmatrix} 0 & -\delta \theta_z & \delta \theta_y \\ \delta \theta_z & 0 & -\delta \theta_x \\ -\delta \theta_y & \delta \theta_x & 0 \end{bmatrix} \mathbf{R} \quad (4)$$

where $\delta \boldsymbol{\theta} = [\delta \theta_x \ \delta \theta_y \ \delta \theta_z]^T$ is a vector of small rotations. Using Eq.(4), Eq.(3) can be rewritten as

$$\delta\lambda_i \mathbf{a}_i + \lambda_i \delta \mathbf{a}_i = \delta \mathbf{C} - \delta \boldsymbol{\theta} \times \mathbf{B}_i - \mathbf{R} \delta \mathbf{B}'_i - \delta \mathbf{A}_{i,0} - l_i \delta \mathbf{n}_i - \delta l_i \mathbf{n}_i \quad (5)$$

where $\mathbf{B}_i$ is represented in the base coordinate frame (i.e., $\mathbf{B}_i = \mathbf{R} \mathbf{B}'_i$). Multiplication of $\mathbf{n}_i^T$ on both sides of Eq.(5) gives

$$\mathbf{n}_i^T \mathbf{a}_i \delta \lambda_i = \mathbf{n}_i^T \delta \mathbf{C} + (\mathbf{n}_i \times \mathbf{B}_i)^T \delta \boldsymbol{\theta} - \mathbf{n}_i^T \mathbf{R} \delta \mathbf{B}'_i - \mathbf{n}_i^T \delta \mathbf{A}_{i,0} - \delta l_i - \mathbf{n}_i^T (\lambda_i \delta \mathbf{a}_i) \quad (6)$$

since $\mathbf{n}_i^T \mathbf{n}_i = 1$ and $\mathbf{n}_i^T \delta \mathbf{n}_i = 0$. Considering all 6 links, Eq.(6) can be represented in a matrix form as

$$\mathbf{J}^{-1} \delta \mathbf{X} = \delta \mathbf{A} + \mathbf{M} \delta \mathbf{L} + \mathbf{N}_1 \delta \mathbf{B}' + \mathbf{N}_2 \delta \mathbf{A}_0 + \mathbf{N}_2 \delta \mathbf{A} \quad (7)$$

where

$$\delta \mathbf{X} = [\delta \mathbf{C} \quad \delta \boldsymbol{\theta}]^T = [\delta X_C \quad \delta Y_C \quad \delta Z_C \quad \delta \theta_x \quad \delta \theta_y \quad \delta \theta_z]^T \quad (8)$$

$$\delta \mathbf{A} = [\delta \lambda_1 \quad \delta \lambda_2 \quad \delta \lambda_3 \quad \delta \lambda_4 \quad \delta \lambda_5 \quad \delta \lambda_6]^T \quad (9)$$

$$\delta \mathbf{L} = [\delta l_1 \quad \dots \quad \delta l_6]^T \quad (10)$$

$$\delta \mathbf{B}' = \begin{bmatrix} \delta \mathbf{B}'_1 \\ \vdots \\ \delta \mathbf{B}'_6 \end{bmatrix} \in \Re^{18 \times 1} \quad (11)$$

$$\delta \mathbf{A}_0 = \begin{bmatrix} \delta \mathbf{A}_{1,0} \\ \vdots \\ \delta \mathbf{A}_{6,0} \end{bmatrix} \in \Re^{18 \times 1} \quad (12)$$

$$\delta \mathbf{A} = \begin{bmatrix} \lambda_1 \delta \mathbf{a}_1 \\ \vdots \\ \lambda_6 \delta \mathbf{a}_6 \end{bmatrix} \in \Re^{18 \times 1} \quad (13)$$

$$\mathbf{J}^{-1} = \begin{bmatrix} \frac{\mathbf{n}_1^T}{\mathbf{a}_1^T \mathbf{n}_1} & \frac{(\mathbf{n}_1 \times \mathbf{B}_1)^T}{\mathbf{a}_1^T \mathbf{n}_1} \\ \vdots & \vdots \\ \frac{\mathbf{n}_6^T}{\mathbf{a}_6^T \mathbf{n}_6} & \frac{(\mathbf{n}_6 \times \mathbf{B}_6)^T}{\mathbf{a}_6^T \mathbf{n}_6} \end{bmatrix} \in \Re^{6 \times 6} \quad (14)$$

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\mathbf{a}_1^T \mathbf{n}_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\mathbf{a}_6^T \mathbf{n}_6} \end{bmatrix} \in \Re^{6 \times 6} \quad (15)$$

$$\mathbf{N}_1 = \begin{bmatrix} \frac{\mathbf{n}_1^T \mathbf{R}}{\mathbf{a}_1^T \mathbf{n}_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\mathbf{n}_6^T \mathbf{R}}{\mathbf{a}_6^T \mathbf{n}_6} \end{bmatrix} \in \Re^{6 \times 18} \quad (16)$$

$$\mathbf{N}_2 = \begin{bmatrix} \frac{\mathbf{n}_1^T}{\mathbf{a}_1^T \mathbf{n}_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\mathbf{n}_6^T}{\mathbf{a}_6^T \mathbf{n}_6} \end{bmatrix} \in \Re^{6 \times 18} \quad (17)$$

Fig. 3 describes an error diagram that shows each kinematic error source's contribution to the pose error of the platform. In this figure, circles denote error boundary about the nominal points, dotted lines denote nominal closed loops, and solid lines denote actual closed loops due to kinematic errors. Note that the terms shown in the figure have the following geometric meanings: $\delta \mathbf{X}$ is the platform pose error vector, $\delta \mathbf{A}$ is an actuator error vector that includes articular variable control and backlash of the gearing, $\delta \mathbf{L}$ is a length error vector of the links due to manufacturing tolerance and imperfect installation between spherical and universal joints, $\delta \mathbf{B}'$ and $\delta \mathbf{A}_0$ are joint installation error vectors of point B'_i and $A_{i,0}$, respectively, and $\delta \mathbf{A}$ is an actuator axis straightness error vector. The errors $\delta \mathbf{L}$, $\delta \mathbf{B}'$, $\delta \mathbf{A}_0$ and $\delta \mathbf{A}$ are called nonactuation errors that can be partly compensated in a controller model by some calibration methods.

If $\mathbf{J}^{-1}$ is invertible, Eq.(7) can be rewritten as

$$\delta \mathbf{X} = \mathbf{E} \delta \boldsymbol{\varepsilon} \quad (18)$$

where

$$\mathbf{E} = \mathbf{J} [\mathbf{I} : \mathbf{M} : \mathbf{N}_1 : \mathbf{N}_2 : \mathbf{N}_2] \in \Re^{6 \times 48} \quad (19)$$

$$\delta\mathbf{e} = [\delta\mathbf{A}^T : \delta\mathbf{L}^T : \delta\mathbf{B}'^T : \delta\mathbf{A}_0^T : \delta\mathbf{A}^T]^T \in \mathbb{R}^{48 \times 1} \quad (20)$$

where $\mathbf{I}$ is the 6×6 identity matrix.

The matrix $\mathbf{E}$ is defined as the total error transformation matrix and the vector $\delta\mathbf{e}$ is defined as the error source vector. Important to note that Eq. (18) provides transformation of all error sources not only through the Jacobian matrix but also through other transformation matrices, $\mathbf{M}$, $\mathbf{N}_1$, and $\mathbf{N}_2$, all depending on the configuration of the manipulator. The diagonal matrices $\mathbf{M}$, $\mathbf{N}_1$, and $\mathbf{N}_2$ imply geometrically that nonactuation errors (length, joint installation, and actuator axis straightness errors) are mapped into actuator errors. This mapping is independently done for each chain because of diagonal matrices. Notice also that when $\mathbf{a}_i^T \mathbf{n}_i = 0$, i.e., when the actuator axis is perpendicular to the link axis, all matrices become singular. Besides, near singular configurations, platform pose errors are dominated more by nonactuation errors than by actuator errors because nonactuation errors are amplified greatly by the $1/\mathbf{a}_i^T \mathbf{n}_i$ factor in $\mathbf{M}$, $\mathbf{N}_1$, and $\mathbf{N}_2$ matrices. Therefore, very accurate kinematic calibration is required for the nonactuation errors.

IV. PERFORMANCE MEASURES ON ACCURACY

This section discusses transformation properties of the total error transformation matrix $\mathbf{E}$ that transforms the error source vector $\delta\mathbf{e}$ to the platform pose error vector $\delta\mathbf{x}$. Based on this discussion, we will deduce some performance measures on accuracy for the optimum architecture design.

To analyze the error output characteristics of the transformation equation, the unit-norm error inputs are often used. However, the unit-norm error inputs may not permit the actual error range of each error source because the maximum errors may significantly vary for different sources. Therefore, individual error sources are normalized using the maximum values of error sources as

$$\begin{aligned} \delta\mathbf{A} &= \|\delta\mathbf{A}\|_{\max} \delta\hat{\mathbf{A}} \\ \delta\mathbf{L} &= \|\delta\mathbf{L}\|_{\max} \delta\hat{\mathbf{L}} \\ \delta\mathbf{B}' &= \|\delta\mathbf{B}'\|_{\max} \delta\hat{\mathbf{B}}' \\ \delta\mathbf{A}_0 &= \|\delta\mathbf{A}_0\|_{\max} \delta\hat{\mathbf{A}}_0 \\ \delta\mathbf{A} &= \|\delta\mathbf{A}\|_{\max} \delta\hat{\mathbf{A}} \end{aligned} \quad (21)$$

where $\odot$ over a vector denotes the normalized vector and $\|\cdot\|_{\max}$ denotes the maximum value of each error source that can be specified by taking control and calibration accuracy into account. Note that the maximum actuator axis straightness error can be defined as $\|\delta\mathbf{A}\|_{\max} = \lambda_{\max} \|\delta\mathbf{a}\|_{\max}$, where $\lambda_{\max}$ is the maximum actuator length. Moreover, since the Jacobian matrix $\mathbf{J}$ is dimensionally inhomogeneous, the concept of characteristic length[4] should be introduced for making dimensionally homogeneous Jacobian matrix $\hat{\mathbf{J}}$.

Then, Eq. (18) can be rewritten as

$$\delta\mathbf{X} = \hat{\mathbf{E}}\delta\hat{\boldsymbol{\varepsilon}} \quad (22)$$

where

$$\begin{aligned} \hat{\mathbf{E}} &= \begin{bmatrix} \|\delta\mathbf{A}\|_{\max} \hat{\mathbf{J}} & \|\delta\mathbf{L}\|_{\max} \hat{\mathbf{J}}\mathbf{M} & \|\delta\mathbf{B}'\|_{\max} \hat{\mathbf{J}}\mathbf{N}_1 & \|\delta\mathbf{A}_0\|_{\max} \hat{\mathbf{J}}\mathbf{N}_2 & \|\delta\mathbf{A}\|_{\max} \hat{\mathbf{J}}\mathbf{N}_2 \end{bmatrix} \\ \delta\hat{\boldsymbol{\varepsilon}} &= \begin{bmatrix} \delta\hat{\mathbf{A}}^T & \delta\hat{\mathbf{L}}^T & \delta\hat{\mathbf{B}}'^T & \delta\hat{\mathbf{A}}_0^T & \delta\hat{\mathbf{A}}^T \end{bmatrix}^T \end{aligned}$$

Consider Eq. (22) as a general transformation[23]:

$$\mathbf{y} = \mathbf{A}\mathbf{x} \quad (23)$$

where $\mathbf{x}$ and $\mathbf{y}$ are $n \times 1$ input and $m \times 1$ output vectors, respectively, and $\mathbf{A}$ is a $m \times n$ transformation matrix. To investigate how the transformation matrix $\mathbf{A}$ affects the magnitude relationship between $\mathbf{x}$ and $\mathbf{y}$, assume that $\mathbf{x}$ lies inside a unit sphere such that

$$\|\mathbf{x}\|^2 \leq 1 \quad (24)$$

then the transformed $\mathbf{x}$ lies inside an ellipsoid defined by

$$\|\mathbf{A}^+\mathbf{y}\|^2 \leq 1 \quad (25)$$

where $\mathbf{A}^+$ is a pseudoinverse of $\mathbf{A}$. The matrix $\mathbf{A}$ can be decomposed into three matrices as

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (26)$$

where $\mathbf{U} = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_m]$ and $\mathbf{V} = [\mathbf{v}_1 \ \cdots \ \mathbf{v}_n]$ are the eigenvector matrices of $\mathbf{A}\mathbf{A}^T$ and $\mathbf{A}^T\mathbf{A}$, respectively, and $\mathbf{\Sigma} = [\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_m) \ \mathbf{0}]$ is a matrix composed of singular values of $\mathbf{A}$ ($\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_m \geq 0$). Using Eq.(26), the pseudoinverse of $\mathbf{A}$ can be written as

$$\mathbf{A}^+ = \mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^T \quad (27)$$

where $\mathbf{\Sigma}^+ = [\text{diag}(\sigma_1^{-1}, \sigma_2^{-1}, \dots, \sigma_m^{-1}) \ \mathbf{0}]^T$ is a pseudoinverse of $\mathbf{\Sigma}$. Inserting Eq.(27) into Eq.(25) gives

$$\begin{aligned} \|\mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^T\mathbf{y}\|^2 &= (\mathbf{y}^T\mathbf{U}\mathbf{\Sigma}^{+T}\mathbf{V}^T)(\mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^T\mathbf{y}) \\ &= (\mathbf{U}^T\mathbf{y})^T \text{diag}(\sigma_1^{-2}, \sigma_2^{-2}, \dots, \sigma_m^{-2})(\mathbf{U}^T\mathbf{y}) \leq 1 \end{aligned} \quad (28)$$

Eq.(28) depicts that $\mathbf{y}$ lies inside an error ellipsoid whose principal axes are $\sigma_1\mathbf{u}_1, \dots, \sigma_m\mathbf{u}_m$.

From the above derivation, the error transformation can be characterized mainly by singular values that can be regarded as error amplification factors (*EAF*). Three different error amplification factors have been considered in the design of robot manipulators; (i) the volume of the error ellipsoid defined in Eq.(28), (ii) the maximum singular value, (iii) the condition number. The volume of the error ellipsoid has similar concept as the manipulability measure proposed by Yoshikawa[24], which is proportional to the ellipsoid volume of a manipulator Jacobian matrix. The maximum singular value can be understood as the maximum error amplification factor. The condition number can be understood as a measure of the relative error amplification of the computed results upon solving a linear system of equations associated with that matrix. This number also characterizes isotropy (shape of the error ellipsoid).

From the total error transformation matrix $\hat{\mathbf{E}}$, three error amplification factors can therefore be defined as

$$EAF_1 = \sqrt{\det(\hat{\mathbf{E}}\hat{\mathbf{E}}^T)} = \prod_{i=1}^6 (\sigma_{\hat{\mathbf{E}}})_i \quad (29)$$

$$EAF_2 = (\sigma_{\hat{\mathbf{E}}})_{\max} \quad (30)$$

$$EAF_3 = (\sigma_{\hat{\mathbf{E}}})_{\max} / (\sigma_{\hat{\mathbf{E}}})_{\min} \quad (31)$$

where $(\sigma_{\hat{\mathbf{E}}})_i$ is the singular value of the total error transformation matrix $\hat{\mathbf{E}}$. The smaller the EAF , the smaller the pose error. Therefore, one of the EAF s can be used as a criterion on accuracy for optimum design problem.

Because all EAF s are pose-dependent, global EAF s over the whole workspace, which are pose-independent, should be used for global optimization of the accuracy. However, in the most actual operation, the manipulator's end-effector does not span the entire workspace because the singular configurations are very close to the workspace boundary. Therefore, a central region around a nominal pose may be considered in design optimization and EAF should be averaged in the central region of the workspace[12]. The averaged global EAF for accuracy criterion can then be defined as

$$GEAF = \frac{\int_{V^*} EAF dV^*}{V^*} \quad (32)$$

where V^* is the volume of the chosen central region.

V. FORMULATION OF THE OPTIMIZATION PROBLEM

In this section, an optimization problem is formulated for the accurate architecture design that reveals minimum platform pose error for given kinematic errors. As discussed in section IV, $GEAF$ is selected as a cost function and is to be minimized. Regarding constraints, we included the desired workspace constraint and limits on the design variables.

To completely determine the HSM geometry, sixty design variables $(\mathbf{A}_{i,0}, \mathbf{A}_{i,1}, \mathbf{B}'_i, l_i)$ should be determined. In order to simplify the design problem, however, we reduced the number of the design variables by exploiting the axis symmetricity of the manipulator. It is assumed that $A_{i,0}$ and $A_{i,1}$ points are located on circles. Fig. 4 shows the reduced set of design variables where R_A and R_B are the radii of the circles passing through $A_{i,0}$ points and B'_i

points, respectively, H is the distance between adjacent $A_{i,0}$ points, α and β are the horizontal and vertical slant angles of the prismatic actuator rail axes, $\lambda_{\max}$ is the maximum actuation length from $A_{i,0}$ to $A_{i,1}$, l is the constant link length, and ψ is the separation angle between adjacent B'_i points. These eight design parameters $(R_A, R_B, H, \alpha, \beta, \lambda_{\max}, l, \psi)$ can completely define the symmetric geometry of the HSM. However, for further simplification, R_A and $\lambda_{\max}$ are predetermined to sufficiently large values that can include translational workspace requirements in our design process. Therefore, the number of final design variables is six. Notice that the platform radius R_B is used as a characteristic length that makes the Jacobian matrix dimensionally homogeneous. For normalization in Eq. (21), we used the maximum error budget in Table 1. Note that the maximum error budget for nonactuation errors can be chosen by considering the maximum allowable error after accurate kinematic calibration. The central region we chose is a parallelepiped (See Fig. 4(b)) whose center lies at one third, from the bottom, of the distance between the upper and the lower limits of the whole translational workspace. In addition, for simplicity of design and analysis, the global *EAF* is averaged only at the center and at each of the eight vertices of the parallelepiped workspace while the orientation of the platform is changing between the maximum and minimum orientation angles at each point. The optimization problem can then be formulated as

$$\begin{aligned}
& \text{Minimize } GEAF \\
& \text{subject to } \begin{array}{l} \text{Design variables limits} \\ \text{Workspace constraints} \end{array}
\end{aligned} \tag{33}$$

The *fmincon* function in MATLAB 5.2 Optimization Toolbox[25] was used to solve the optimization problem. This minimization function uses a Sequential Quadratic Programming (SQP) method, in which a Quadratic Programming (QP) subproblem is solved at each iteration. In this minimization algorithm, an estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula.

VI. OPTIMIZATION RESULTS

For three *EAFs* as discussed in section IV, Fig. 5 shows the initial and optimized architectures of the HSM, while Table 2 shows the numerical values of the design parameters. The corresponding translational workspaces are shown in Fig. 6. As are seen in those figures and Table 2, the error ellipsoid volume and the maximum singular value criteria produced similar optimal architectures and workspaces while the condition number criterion generated

different optimal architecture (Design parameters for H and α are at their minimum limits and R_B has a smaller value than that from other two criteria).

In order to estimate how much the pose error of the platform is reduced from the initial design by the proposed optimization method, the manipulator's accuracy has been evaluated by the Monte Carlo statistical experiment approach[21]. In the Monte Carlo simulation, we generated random vectors for $\delta\mathbf{B}'$, $\delta\mathbf{A}_0$, $\delta\mathbf{A}$ with uniform error distribution inside an error sphere with the center at the nominal location of a point and with a radius that is the maximum allowable installation error, say 10 μm radius (See Fig. 7). For $\delta\mathbf{A}$ and $\delta\mathbf{L}$, we assumed a maximum allowable steady state control error and a link manufacturing tolerance. All of these length errors are assumed to follow uniform distribution with maximum limits specified by an error budget guide in Table 1. The resultant pose errors are determined by the error model in Eq.(18). The average pose error over the central parallelepiped workspace is obtained at the points where the objective function was averaged. The average value is obtained by 5000 random values for meaningful Monte Carlo simulation[21]. We selected three times of standard deviation as the maximum pose error that are shown in Table 3 for the initial and the optimized design.

From Table 3, the following results can be observed; (i) the position errors in the X and Y directions are significantly reduced by about 40-56% for all EAF s, while the position error in Z direction is slightly reduced (about 9-17%). (ii) The orientation errors in all directions are significantly reduced by about 80% for all EAF s. (iii) The error ellipsoid volume and the maximum singular value criteria generated less orientation errors than position errors and showed similar results in terms of level of accuracy improvement. Meanwhile, the condition number criterion reduced position errors more than orientation errors.

VII. CONCLUSIONS

In this paper, an optimal accuracy design based on the volumetric error model had been performed for a HexaSlide manipulator. Possible kinematic errors were identified and geometrically interpreted from the differential error model and the pose error of the platform was obtained from the kinematic error sources through the total error transformation matrix. Then using the error equation, three different error amplification factors were used as design criteria for accuracy. A design optimization of the HSM had been performed by minimizing the error amplification factors with considering the workspace and design variable limits. Finally the pose error was evaluated by Monte Carlo simulation technique and it was shown that the optimized manipulator had substantially smaller errors

compared with the initial design.

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(a) Overview

(b) Schematic Diagram

Fig. 1 HexaSlide Parallel Manipulator

Fig. 2 Schematic Diagram of One Chain

Fig. 3 Error Diagram

Fig. 4 (a) Design Variables, (b) Workspace constraint

(a) Initial Design

(b) Volume

(c) Max. Singular Value

(d) Condition No.

Fig. 5 Initial and Optimum Architectures

(a) Initial Design

(b) Volume

(c) Max. Singular Value

(d) Condition No.

Fig. 6 Translational Workspaces at Initial and Optimum Designs

Fig. 7 Allowable Installation Error Sphere

TABLE 1 MAXIMUM ERROR BUDGET (μm)

$ \delta\lambda _{\max}$	$ \delta l _{\max}$	$\ \delta\mathbf{B}'\ _{\max}$	$\ \delta\mathbf{A}_0\ _{\max}$	$\ \delta\mathbf{A}\ _{\max}$
1	10	10	10	10

TABLE 2 INITIAL AND OPTIMUM DESIGN VALUES

	Initial	Optimum		
		Volume	Max.Sinular Value	Condition No.
H (mm)	220.0	208.35	415.26	70.000
β ($^\circ$)	30.00	56.631	59.348	53.285
ψ ($^\circ$)	83.60	105.56	105.56	105.56
l (mm)	992.8	960.05	775.70	868.71
R_b (mm)	165.0	655.12	642.99	293.25
α ($^\circ$)	0	27.210	23.865	0.0000

Design Variable Limits :

$$70 \leq H, \quad \beta \leq 90, \quad 28.888 \leq \psi \leq 105.56, \quad 0 \leq l, \quad 73.612 \leq R_b, \quad -180 \leq \alpha \leq 180$$

TABLE 3 ACCURACY COMPARISON

	Initial	Optimum		
		Volume	Max.Sinular Value	Condition No.
δx (μm)	52.470	31.662	28.851	22.657
δy (μm)	51.650	29.188	27.271	22.092
δz (μm)	14.328	11.842	12.467	13.043
$\delta\theta_x$ ($^\circ$)	7.3796×10^{-3}	1.4360×10^{-3}	1.4629×10^{-3}	3.4446×10^{-3}
$\delta\theta_y$ ($^\circ$)	8.9670×10^{-3}	1.3080×10^{-3}	1.4490×10^{-3}	3.7522×10^{-3}
$\delta\theta_z$ ($^\circ$)	12.936×10^{-3}	1.9345×10^{-3}	1.8152×10^{-3}	3.1194×10^{-3}