State Estimation and Parameter Identification of Flexible Manipulators Based on Visual Sensor and Virtual Joint Model

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Abstract

When we control a flexible manipulator based on a dynamic model, we need to estimate the state variables with accuracy. In the case of rigid manipulators, the state variables consist of joint angles and their velocities, which are measured easily by encoders, potentiometers, tachometers, and so on. In the case of flexible manipulators, however, the state variables also include elastic deformations and their velocities due to flexibility.

Here we focus on visual sensor to measure distributed state variables of flexible manipulators. First, we discuss a distributed state estimation method which uses discrete position information of markers attached on flexible links. Second, when we apply the virtual joint model to flexible manipulators, elastic deformations are expressed by virtual passive joint angles which are lumped. So we have to correspond the real elastic deformations to the virtual joint angles. We also discuss this transformation. Third, when we estimate the lumped state variables by the proposed method, it is possible to identify physical parameters of the dynamic model. Finally, some experiments were carried out to verify the effectiveness of the proposed method.

1 Introduction

Generally, for precise control or analysis of dynamic motion of a flexible manipulator, it is essential to have a dynamic model of flexibility [1, 2, 3]. As one of the modeling methods of flexible manipulators, the virtual joint model were proposed by Yoshikawa *et al.* [4]. Parameter identification and manipulator control based on the virtual joint model have been studied [4, 5].

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Figure 1: Flexible manipulator

In a previous research [5], the state observer was used for state estimation of this dynamic model. However, precise state estimation by this method becomes difficult when the virtual link division number is large, which is the case that the links have much more flexibility.

In this paper, we propose a state estimation method based on discrete position information of flexible links by visual sensor. Some previous research [6] used visual sensors to control flexible manipulators, but the visual sensors were utilized just to measure the tip position of flexible link, not to estimate the state variables directly.

2 Flexible Manipulator

We consider a flexible manipulator shown in **Fig.1** in this paper. It is N d.o.f. plane manipulator which has serial N flexible links and N joints, and fixed on the inertial coordinate system Σ_0 .

Each joint displacement is $\theta = [\theta_1, \theta_2, \dots, \theta_N]^T$. The length of link i ($i = 1, 2, \dots, N$) without elastic deformation is l_i , and the mass is m_i .

Figure 2: Markers on the flexible link i

Figure 3: Flexible link shape function

The link i coordinate system Σ_i is fixed on the center of joint i . Its x_i axis is along link i when it's not deformed, z_i axis is along the displacement axis of joint i , and y_i axis is determined to compose the right hand coordinate system.

3 Distributed State Estimation Based on Visual Sensor

In this section, we describe a method utilizing a visual sensor to estimate distributed state variables of flexible manipulators.

In this method, only visual markers are attached on flexible links, so the system of manipulator side becomes simple compared to other methods, such as strain gages or accelerometers. The visual sensor measures position information, which is not differential value, we don't have to consider its integration error. If more precision is needed, we can increase the points of measurement easily. If two or more cameras are available, we can get 3-dimensional information of markers to extend this method.

3.1 Attaching Markers

In this method, k markers are attached on link i as shown in **Fig.2**. As we mention in following Section 3.2 and 3.3, it is essential the number of markers enough to determine a flexible link shape function.

If we know the minimum wave length λ_{min} which composes the link shape by physical consideration such

as natural frequencies, the minimum k is constrained by the sampling theorem.

$$k \geq \frac{2l_i}{\lambda_{min}} \quad (1)$$

3.2 Flexible Link Shape Function

The elastic deformation $y_i(x_i, t)$ of link i and its velocity $v_i(x_i, t)$ at time t are distributed state variables (see **Fig.3**).

We choose a set of approximation functions to estimate this state variables from discrete position information of markers. We call this set the flexible link shape function. Here we adopt polynomial approximation as the link shape function :

$$y_i(x_i, t) = \sum_{j=1}^{o_{max}} a_{ij}(t) x_i^j \quad (2)$$

where o_{max} is the maximum order of the polynomial.

This o_{max} may cause improper approximation if it is too large. In many cases o_{max} must be smaller than 4 or 5, though it depends on the marker number k . If the link is so flexible that o_{max} order polynomial does not reflect the real link shape, we should adopt another one as the link shape function, for instance, trigonometric function.

3.3 Fitting Link Shape Function

Consider the polynomial function (2). When the position information of k markers on link i

$${}^i\mathbf{p}_j(t) = \begin{bmatrix} {}^i p_{xj}(t) \\ {}^i p_{yj}(t) \end{bmatrix} \quad (j = 1, 2, \dots, k) \quad (3)$$

is measured by a visual sensor, the measured values matrix ${}^i\mathbf{P}(t)$, the measured values vector ${}^i\mathbf{Y}(t)$, and coefficients vector of the link shape function $\mathbf{a}_i(t)$ are defined as follows.

$${}^i\mathbf{P}(t) = \begin{bmatrix} {}^i p_{x1} & {}^i p_{x1}^2 & \cdots & {}^i p_{x1}^{o_{max}} \\ {}^i p_{x2} & {}^i p_{x2}^2 & \cdots & {}^i p_{x2}^{o_{max}} \\ \vdots & \vdots & \ddots & \vdots \\ {}^i p_{xk} & {}^i p_{xk}^2 & \cdots & {}^i p_{xk}^{o_{max}} \end{bmatrix} \quad (4)$$

$${}^i\mathbf{Y}(t) = \begin{bmatrix} {}^i p_{y1} & {}^i p_{y2} & \cdots & {}^i p_{yk} \end{bmatrix}^T \quad (5)$$

$$\mathbf{a}_i(t) = \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{io_{max}} \end{bmatrix}^T \quad (6)$$

Then the coefficients vector $\mathbf{a}_i(t)$ are estimated by the least square method.

$$\mathbf{a}_i(t) = \{({}^i\mathbf{P})^T({}^i\mathbf{P})\}^{-1}({}^i\mathbf{P})^T({}^i\mathbf{Y}) \quad (7)$$

Figure 4: Construction of virtual joint model ($n = 3$)

4 Virtual Joint Model

In this section, we describe a model construction of flexible manipulators based on the virtual joint model by Yoshikawa *et al.* [4]

4.1 Model Construction

In the virtual joint model, which is one of the lumped parameter models, each flexible link is divided into some virtual rigid links connected by virtual passive joints. The link flexibility is represented by virtual springs and dampers attached to the virtual passive joints.

As shown in **Fig.4**, each of N flexible links is divided by n virtual passive joints. The divided virtual rigid links are numbered link $i0, i1, \dots, in$ from the manipulator base (**Fig.4** is an example of $n = 3$). l_{ij} is the length, l_{gij} is the center of mass, m_{ij} is the mass, I_{ij} is the inertia moment around the center of mass of link ij ($j = 0, 1, \dots, n$).

The virtual passive joint attached to the base of link ij ($j = 1, 2, \dots, n$) is called joint ij . The displacement of the virtual passive joints on link i is $\phi_i(t) = [\phi_{i1}, \phi_{i2}, \dots, \phi_{in}]^T$. The damping coefficient and spring constant of joint ij are respectively D_{ij} , K_{ij} .

4.2 Dynamics

The equations of motion of a flexible manipulator based on the virtual joint model are given by

$$\begin{aligned} M_{11}(\psi)\ddot{\theta} + M_{12}(\psi)\ddot{\phi} + h_1(\psi, \dot{\psi}) &= \tau \\ M_{21}(\psi)\ddot{\theta} + M_{22}(\psi)\ddot{\phi} + h_2(\psi, \dot{\psi}) + D\dot{\phi} + K\phi &= 0 \end{aligned} \quad (8)$$

M_{11} , M_{12} , M_{21} , M_{22} are the inertia matrices, h_1 , h_2 are the vectors containing centrifugal, Coriolis, gravity and other nonlinear force, $D = \text{diag}[D_{11}, D_{12}, \dots, D_{Nn}]$, $K =$

Figure 5: Fitting into virtual joint model ($n=3$)

$\text{diag}[K_{11}, K_{12}, \dots, K_{Nn}]$ are respectively the diagonal matrices of damping coefficients and spring constants. τ is the joint driving force. $\phi = [\phi_1^T, \phi_2^T, \dots, \phi_N^T]^T$ is the vector of passive joint angles, $\psi = [\theta^T, \phi^T]^T$ is the vector of all joint angles.

5 Lumped State Estimation of Virtual Joint Model

In this section, we describe a method to transform the distributed state variables of flexible links into lumped state variables of the virtual joint model. When we get the lumped state variables by the proposed method, it is also possible to identify physical parameters of the dynamic model.

5.1 Transformation into Lumped State Variables

Here we set the position of virtual passive joints and link tip as the representative points of link i . ${}^i\hat{p}_1(t), \dots, {}^i\hat{p}_{n+1}(t)$ denote the position of representative points of link i .

As shown in **Fig.5**, the representative point ${}^i\hat{p}_j(t) = [{}^i\hat{p}_{xj}, {}^i\hat{p}_{yj}]^T$ ($j = 2, \dots, n+1$) is determined to locate on the link i shape function $y_i(x_i, t)$. First, ${}^i\hat{p}_1(t)$ is given by

$${}^i\hat{p}_1(t) = \begin{bmatrix} l_{i0} \\ 0 \end{bmatrix} \quad (9)$$

then, ${}^i\hat{p}_j(t)$ ($j = 2, \dots, n+1$) is determined as the point satisfying the following simultaneous equations.

$${}^i\hat{p}_{yj} = y_i({}^i\hat{p}_{xj}, t) \quad (10)$$

$$({}^i\hat{p}_{xj} - {}^i\hat{p}_{x,j-1})^2 + ({}^i\hat{p}_{yj} - {}^i\hat{p}_{y,j-1})^2 = l_{i,j-1}^2 \quad (11)$$

From these representative points, the virtual pas-

sive joint angles $\phi_i(t)$ are given by

$$\phi_{ij}(t) = \tan^{-1} \left(\frac{i\hat{p}_{y,j+1} - i\hat{p}_{yj}}{i\hat{p}_{x,j+1} - i\hat{p}_{xj}} \right) - \sum_{j'=1}^{j-1} \phi_{ij'} \quad (12)$$

The virtual passive joint velocity $\dot{\phi}_i(t) = [\dot{\phi}_{i1}, \dot{\phi}_{i2}, \dots, \dot{\phi}_{in}]^T$ is estimated by numerical differentiation of $\phi_i(t)$ with respect to time t .

The transformation matrix ${}^{i+1}\mathbf{T}_i$ from Σ_i to Σ_{i+1} is also obtained from $\phi_i(t)$.

5.2 Parameter Identification

In the equations of motion (8), $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t)$, $\tau(t)$ can be measured by sensors of real manipulator. $\phi(t)$, $\dot{\phi}(t)$, $\ddot{\phi}(t)$ can be obtained by the proposed method and numerical differentiation. From this set of values, it is possible to identify the virtual joint model parameters α defined by

$$\alpha = [l_{10}, \dots, l_{Nn}, l_{g10}, \dots, l_{gNn}, m_{10}, \dots, m_{Nn}, I_{10}, \dots, I_{Nn}, D_{11}, \dots, D_{Nn}, K_{11}, \dots, K_{Nn}]^T \in \mathbb{R}^{6Nn+4} \quad (13)$$

For any given value of α , the following values can be calculated.

$$\tau_{ea} = \mathbf{M}_{11}(\psi)\ddot{\theta} + \mathbf{M}_{12}(\psi)\ddot{\phi} + \mathbf{h}_1(\psi, \dot{\psi}) - \tau \quad (14)$$

$$\tau_{ep} = \mathbf{M}_{21}(\psi)\ddot{\theta} + \mathbf{M}_{22}(\psi)\ddot{\phi} + \mathbf{h}_2(\psi, \dot{\psi}) + \mathbf{D}\dot{\phi} + \mathbf{K}\phi$$

$\tau_{ea}(\alpha, t)$ and $\tau_{ep}(\alpha, t)$ are the torque errors between the real manipulator and the model for the active and passive joints, respectively.

The performance function $J(\alpha)$ is defined in a time period $t_i \leq t \leq t_f$ as

$$J(\alpha) = \int_{t_i}^{t_f} (\tau_{ea}^T \tau_{ea} + \tau_{ep}^T \tau_{ep}) dt \quad (15)$$

The model parameters α are optimized by solving the following nonlinear programming problem.

$$\text{minimize : } J = J(\alpha) \quad (16)$$

$$\text{subject to : } 0 \leq l_{i0} \leq l_{i0max} \quad (17)$$

$$l_{ijmin} \leq l_{ij} \quad (j = 1, 2, \dots, n) \quad (18)$$

$$0 < m_{ij}, I_{ij}, D_{ij}, K_{ij} \quad (19)$$

$$\sum_{j=0}^n l_{ij} = l_i, \quad \sum_{j=0}^n m_{ij} = m_i \quad (20)$$

where l_{i0max} and l_{ijmin} are certain positive constants to constrain the virtual link lengths.

Figure 6: Experimental system

6 Experiment

In this section, the experiment of state estimation and parameter identification is shown to verify the proposed method.

6.1 Experimental System

The system overview is shown in **Fig.6**. The flexible link is a flat cantilever beam made of stainless steel. Its length is 1.00 [m], the width is 35.0 [mm], the thickness is 1.5 [mm] and the mass is 0.44 [kg]. The picture of this flexible link is shown in **Fig.7**.

Five LEDs (their diameters are 1.0 [mm]) are attached on the link as visual markers. The positions of markers are 0.20, 0.40, 0.60, 0.80, 1.00 [m] from the proximal end of the link. The motion of the LEDs is taken by the CCD camera TI-24A of NEC Co. Ltd. This screen image is sent to the video tracker G2120II of OKK Co. Ltd. and the positions of the markers are measured. The sampling frequency of the video tracker is 30 [Hz]. These position data are calculated by the PC9821 Xa12 (Pentium 120MHz) of NEC Co. Ltd.

An accelerometer is attached on the link tip. The acceleration data is analyzed by the FFT analyzer CF-1200 of ONO SOKKI Co. Ltd. This information

Figure 7: Flexible link

Figure 8: Position measurements of the markers on the flexible link (+ : $x = 0.2$ [m], $\circ$: $x = 0.4$ [m], * : $x = 0.6$ [m], $\times$: $x = 0.8$ [m], $\diamond$: $x = 1.0$ [m], when without deformation)

is used only for the verification of identified dynamic model.

6.2 Measured Data

From a stationary state with a certain displacement, an appropriate vibration is applied at $t = 0$ [s] and the displacement data in the directions x and y of the markers are measured. Two example data are shown in **Fig.8** (only y direction data).

6.3 State Estimation and Parameter Identification

The virtual passive joint angles are estimated and the virtual joint model is identified by the proposed method using the 1st mode dominant vibration data of **Fig.8** (a).

The estimated virtual passive joint angle data are rather rough because of the sampling frequency of the video tracker. So these angles are interpolated every 4 [ms] by cubic spline curve to obtain the angular velocities and the angular accelerations.

In the model identification, the virtual passive joint number n is 2, in order to express the 1st and 2nd mode of vibration.

The parameters to be identified are reduced to the

Figure 9: State estimation from the experimental data (* : marker position from visual sensor, thin line : link shape function, $\circ$: estimated virtual joint position, thick line : estimated virtual rigid link)

link lengths, damping coefficients, and spring constants. Because this flexible link is a flat cantilever, other parameters can be calculated by the link lengths.

On these settings, the parameter of virtual joint model is obtained as shown in **Table 1**.

Fig.9 is the example of the state estimation from the data **Fig.8** (a) using the virtual joint model of **Table 1**. In this figure, the results from $t = 0.0$ to 0.4 [s] with time interval 0.1 [s] are shown.

6.4 Verification of Result

The natural frequencies of the virtual joint model of **Table 1** are calculated and compared with the real

Table 1: Identified parameters of the flexible beam

(a) Virtual rigid links				
Link j	Length l_j [m]	Mass center l_{gj} [m]	Mass m_j [kg]	Inertia I_j [kg·m ²]
0	0.000	0.000	0.000	0.0
1	0.568	0.284	0.250	6.72×10^{-3}
2	0.432	0.216	0.190	2.96×10^{-3}
Total	1.000	—	0.440	—

(b) Virtual passive joints		
Joint j	Spring constant k_j [N·m/rad]	Damping coefficient d_j [N·m·s/rad]
1	12.1	1.00×10^{-2}
2	4.24	1.00×10^{-3}

Figure 10: Simulation results based on identified parameters (solid line : simulation results, $\diamond$: real flexible link tip)

measured values in **Table 2**.

For the case of 1st or 2nd mode dominant vibration shown in **Fig.8**, the tip point behavior of the real flexible link and the virtual joint model are compared on the same initial conditions. These results are shown in **Fig.10**. In the simulation, the Runge-Kutta method (every 100 μ s) is used for numerical integration.

6.5 Discussion

From the simulation result in **Fig.10**, the real flexible link and the simulation show a tendency to fit the behaviors generally.

As can be seen from **Table 2**, however, the natural frequencies are a little different. This difference causes the phase shift in the simulation.

For the original identification method of the virtual joint model, this kind of phase shift doesn't occur [5]. The original method [4, 5] includes the natural fre-

quency errors in the performance function J . On the other hand, the proposed method here doesn't include natural frequency errors directly. This problem may cause the phase shift in the simulation.

Consequently, the performance function J (15) can include the natural frequency errors for more precision, if those information are available.

7 Summary and Conclusion

In this research, we proposed a state estimation method and a parameter identification method of the dynamic model of flexible manipulators based on the virtual passive joint model. We verified the effectiveness of the proposed method by the experiment.

The intended purpose of this state estimation method is to control the vibration of flexible manipulators. In the near future, we want to apply this method to the precise control based on the virtual joint model.

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Table 2: Comparison of natural frequencies

	1st [Hz]	2nd [Hz]
real flexible link	1.25	7.25
virtual joint model	1.34	6.02