

Furthermore, we introduce two application-specific parameters, which are useful for the study of stereo panorama cameras: (1) the distance, denoted as H_1 , between camera focal point C and target range of scene objects of interest (e.g. \mathbf{P}_1 in Fig. 1); and (2) the width of the angular disparity interval, denoted as θ_w , defined by the difference between minimum and maximum angular disparities² in a resultant stereo panoramic pair. The parameter H_1 influences the pictorial composition via a ‘factoring’ of the vertical field of view. The parameter θ_w determines stereo acuity. Both parameters can be used in image acquisition to formulate constraints for the relations between camera-specific parameters and scene range descriptors, and are usually calculated by the application requirement.

Note that the parameters introduced can all be orthographically projected onto the camera’s focal plane on which all the camera’s focal points lie. Without loss of validity, the following studies/analyses are presented in two-dimensional space.

3. Problem Statement

The specifications of application requirements for stereo panorama image acquisition can be described by intervals of scene range descriptors and application-specific parameters. Formally, we define them as $[D_{1min}, D_{1max}]$, $[D_{2min}, D_{2max}]$, $[H_{1min}, H_{1max}]$, and $[\theta_{wmin}, \theta_{wmax}]$, where $D_{1max} < D_{2min}$.

In theory, the value of R can be any positive real and the value of ω can be any positive real less than 360° . Practically, motivated by system realization or cost issues, the intervals of both parameters should be as small as possible for a given application.

Without loss of generality, let $R_{min} = \omega_{min} = 0$, and consider the problem of finding minimum values of R_{max} and ω_{max} that fully satisfy the specifications of application requirements.

4. Analysis

Geometrically for each of parameters R and ω the problem consists in finding the maximum value in a bounded four-dimensional space. It is difficult to imagine such a hyper-surface in a five-dimensional space. To understand the behavior of such a hyper-surface in our case, we analyze individual relations between values of R and ω and values of all other camera parameters.

In Fig. 2, we illustrate how the values of R and ω change as the value of one of the scene parameters changes while

²Angular disparity is defined by the angle between two rays, starting at rotation center \mathbf{O} and passing through a pair of corresponding projections of a 3D point.

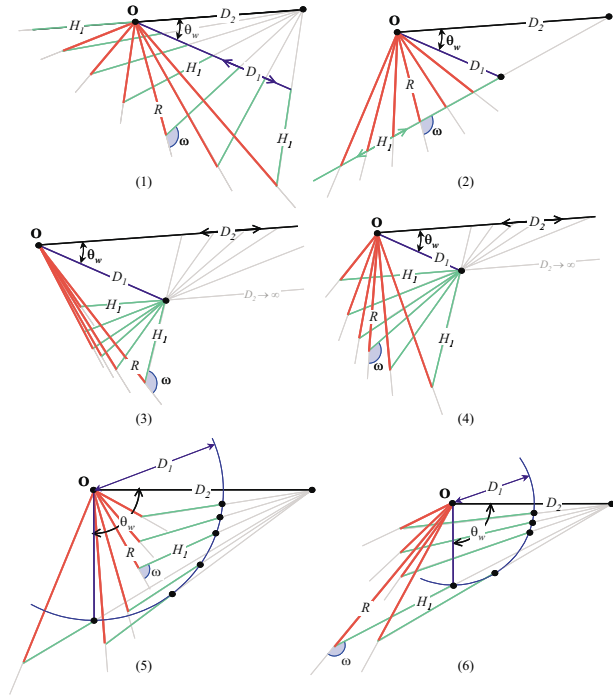


Figure 2. Analysis of the geometric relations.

all others remain constant. In each drawing, we only show a few states of the particular variable to demonstrate the changing behaviors of R and ω values. In each case, if different constant values are chosen, only the magnitudes of the R and ω values will change accordingly, but the ‘behaviors’ of their changes will remain the same.

4.1. D_1 vs. R and ω

For some constant values of D_2 , H_1 , and θ_w , the geometry of a change of the D_1 value versus changes in R and ω values is visualized in Fig. 2(1). From such geometric studies we conclude the following:

- (i) we have $\lim_{D_1 \rightarrow 0^+} R = H_1$ and $\lim_{D_1 \rightarrow 0^+} \omega = 180^\circ$;
- (ii) we have

$$\lim_{D_1 \rightarrow D_2^-} R = \sqrt{D_2^2 + H_1^2 + 2D_2H_1 \sin\left(\frac{\theta_w}{2}\right)}, \text{ and}$$

$$\lim_{D_1 \rightarrow D_2^-} \omega = \arccos\left(\frac{-H_1 - D_2 \sin\left(\frac{\theta_w}{2}\right)}{\sqrt{D_2^2 + H_1^2 + 2D_2H_1 \sin\left(\frac{\theta_w}{2}\right)}}\right);$$

- (iii) by (i) and (ii), we know that $\lim_{D_1 \rightarrow 0^+} R < \lim_{D_1 \rightarrow D_2^-} R$ and $\lim_{D_1 \rightarrow 0^+} \omega > \lim_{D_1 \rightarrow D_2^-} \omega$;

and these relations are valid for any given values of D_2 , H_1 , and θ_w . For all the values of D_1 in interval $(0, D_2)$, it also follows that there exist values of R and ω less than the limits calculated above respectively.

4.2. H_1 vs. R and ω

For some constant values of D_1 , D_2 , and θ_w , the geometry of changing the H_1 value versus changes in R and ω values is visualized in Fig. 2(2). From these geometric studies it follows:

- (i) we have $\lim_{H_1 \rightarrow 0^+} R = D_1$ and

$$\lim_{H_1 \rightarrow 0^+} \omega = \arccos \left(\frac{D_2 \cos(\theta_w) - D_1}{\sqrt{D_1^2 + D_2^2 - 2D_1 D_2 \cos(\theta_w)}} \right);$$

- (ii) we have $\lim_{H_1 \rightarrow \infty} R = \infty$ and $\lim_{H_1 \rightarrow \infty} \omega = 180^\circ$;

and these relations are valid for any given values of D_1 , D_2 , and θ_w . For all the values of H_1 in $(0, \infty)$, it also follows:

- (i) there exist values of R less than $\lim_{H_1 \rightarrow 0^+} R$;
(ii) the value of ω increases/decreases while the value of H_1 increases/decreases.

4.3. D_2 vs. R and ω

Figure 2(3) and (4), show the geometry of changing the D_2 value versus changes in R and ω values when the values of D_1 , H_1 , and θ_w are kept constant. In (3) we have the case when $D_1 \geq H_1$, and in (4) we have $D_1 < H_1$. For any given values of D_1 , H_1 , and θ_w , we have

$$\lim_{D_2 \rightarrow D_1^+} \omega = \arccos \left(\frac{-H_1 - D_1 \sin\left(\frac{\theta_w}{2}\right)}{\sqrt{D_1^2 + H_1^2 + 2D_1 H_1 \sin\left(\frac{\theta_w}{2}\right)}} \right),$$

and

$$\lim_{D_2 \rightarrow \infty} \omega = 180^\circ - \arccos \left(\frac{H_1 - D_1 \cos(\theta_w)}{\sqrt{D_1^2 + H_1^2 - 2D_1 H_1 \cos(\theta_w)}} \right).$$

All values of D_2 in (D_1, ∞) satisfy the following relations:

- (i) In both cases, the value of R increases/decreases while the value of D_2 decreases/increases.
(ii) In case (3), the value of ω increases/decreases while the value of D_2 decreases/increases.
(iii) In case (4), there exist values of ω less than both $\lim_{D_2 \rightarrow D_1^+} \omega$ and $\lim_{D_2 \rightarrow \infty} \omega$.

4.4. θ_w vs. R and ω

The valid value of θ_w is defined to be in the open interval $(0^\circ, 90^\circ)$. Figure 2(5) and (6) show the geometry of changing the θ_w value versus changes in R and ω values when the values of D_1 , D_2 , and H_1 are kept constant. In (5) we have the case that $D_1 \geq H_1$, and in (6) we have $D_1 < H_1$. These geometric studies prove:

- (i) in case (5), we have $\lim_{\theta_w \rightarrow 0^+} \omega = 0^\circ$, and in case (6), we have $\lim_{\theta_w \rightarrow 0^+} \omega = 180^\circ$;
(ii) in both cases, we have

$$\lim_{\theta_w \rightarrow 90^-} \omega = \arccos \left(\frac{-D_1^2}{\sqrt{A(D_1^2 + H_1^2) + 2D_1^2 H_1 \sqrt{A}}} \right),$$

$$\text{where } A = (D_1^2 + D_2^2);$$

for any given values of D_1 , D_2 , and H_1 . For all the values of θ_w in $(0^\circ, 90^\circ)$ it follows:

- (i) in both cases, the value of R increases/decreases while the value of D_2 increases/decreases;
(ii) in case (5), the value of ω increases/decreases while the value of D_2 increases/decreases;
(iii) in case (6), there exist values of ω less than both $\lim_{\theta_w \rightarrow 0^+} \omega$ and $\lim_{\theta_w \rightarrow 90^-} \omega$.

5. Camera Parameters Design

This section explains how the camera parameters R and ω are designed using previous analysis results.

The individual graphs of R and ω with respect to each of the parameters D_1 , H_1 , D_2 , and θ_w are shown in Fig. 3. All graphs illustrate the general behavior of how the values of R and ω change while just one of the values of D_1 , H_1 , D_2 , and θ_w varies.

Let functions $f_1(D_1)$, $f_2(H_1)$, $f_3(D_2)$, and $f_4(\theta_w)$ be defined to be the value of R with respect to the single variable D_1 , H_1 , D_2 , and θ_w respectively. Similarly, functions $f_5(D_1)$, $f_6(H_1)$, $f_7(D_2)$, and $f_8(\theta_w)$ are defined to be the value of ω following the same convention. All of them are continuous functions.

Both functions $f_1(D_1)$ and $f_2(H_1)$ have a single minimum, and their graphs are concave upward on $(0, D_2)$ and $(0, \infty)$ respectively. Functions $f_3(D_2)$ and $f_4(\theta_w)$ are decreasing on (D_1, ∞) and $(0^\circ, 90^\circ)$ respectively. Function $f_5(D_1)$ has a single minimum on $(0, D_2)$. Function $f_6(H_1)$ is increasing on $(0, \infty)$. Moreover, in the case of $D_1 \geq H_1$, function $f_7(D_2)$ is decreasing on (D_1, ∞) . On the other hand, in the case of $D_1 < H_1$, function $f_7(D_2)$ has a single minimum on (D_1, ∞) . Finally, in the case of $D_1 \geq H_1$, function $f_8(\theta_w)$ is increasing on $(0^\circ, 90^\circ)$. For the case of $D_1 < H_1$, function $f_8(\theta_w)$ has a single minimum on $(0^\circ, 90^\circ)$.

Since all the graphs of functions f_1 , f_2 , f_3 , and f_4 are decreasing, increasing or concave upward, we may conclude that the maximum value of R within the bounded region defined by intervals $[D_{1min}, D_{1max}]$, $[D_{2min}, D_{2max}]$, $[H_{1min}, H_{1max}]$, and $[\theta_{wmin}, \theta_{wmax}]$ lies on the boundary of the region. This reduces the search space for the maximum of R from a four-dimensional bounded space down to

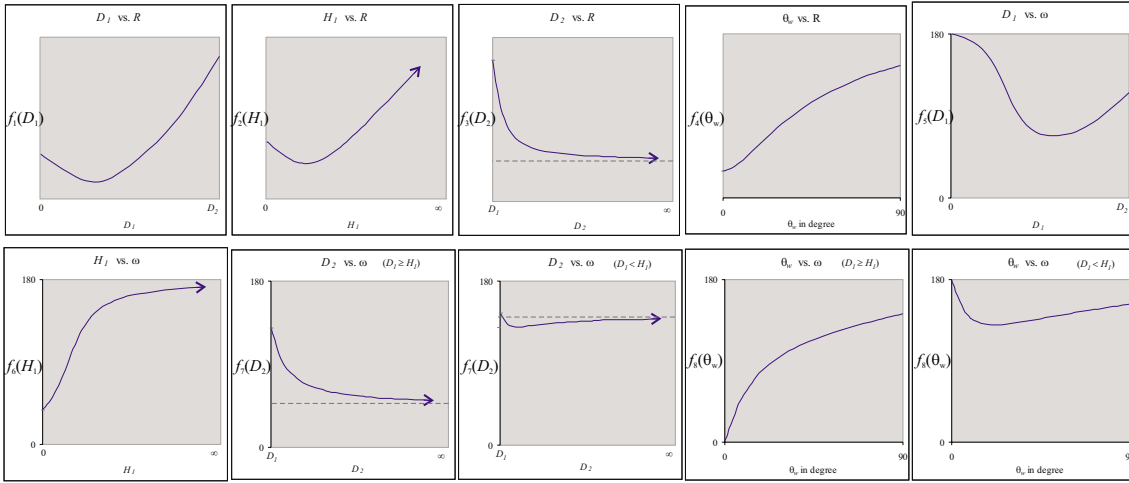


Figure 3. The graphs of R and ω with respect to each of the parameters.

16 (check/search) points, i.e. all possible combinations of these boundary values.

Furthermore, since function $f_3(D_2)$ is decreasing, the maximum value of R exists when D_2 equals to D_{2min} . Since $f_4(\theta_w)$ is an increasing function, the maximum value of R exists when θ_w equals to θ_{wmax} . From these results, we may conclude that the search space of 16 points can be further reduced to four to find the maximum value of R , i.e. the minimum R for intervals defined by application-specific requirements.

Similar to functions f_1, f_2, f_3 , and f_4 all the graphs of functions f_5, f_6, f_7 , and f_8 are decreasing, increasing or continuous and have a single minimum, we may again conclude that the maximum value of ω also lies on the boundary of the region. This reduces the search space for the maximum of ω to also 16 (check/search) points.

Seeing as $f_6(H_1)$ is an increasing function, the maximum value of ω exists when H_1 equals to H_{1max} . This further reduces the search space from 16 points to eight for finding the maximum value of ω . Moreover, if $D_{1min} \geq H_{1max}$, then only one comparison between two values is sufficient to find the maximum value of ω . Alternatively, if $D_{1min} \leq H_{1max} \leq D_{1max}$, then the cardinality of our search space is just five.

6. Conclusions

The acquisition of stereo panoramas requires dynamic adjustments for different scene ranges. Stereo panoramic camera design should incorporate such commonly demanded functions such that high quality of produced stereo panoramas for dynamic ranges of scenes can be achieved. This paper addressed a novel approach to camera parameter design for an already known (and commercially avail-

able) architecture of stereo panorama cameras, and ensures that the requirements of desired pictorial compositions and stereo acuity over specified dynamic scene ranges are attainable. Moreover, our approach contributes to the design process for camera parameters by reducing the search space drastically from a four-dimensional bounded set to seven comparisons at most.

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