On the Selection of Code Set for Binary Multi-code CDMA System

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Abstract. Multi-code CDMA system provides high speed and variable rate services with different quality requirements. Since envelope fluctuation of the multi-code signal is large, the system requires highly linear power amplifier. In this paper, we consider a binary level multi-code pulse width (PW-) CDMA system. In the system, linear sum of orthogonal codes is truncated and pulse width modulated. Since the transmitting signal has constant envelope, and the receiver structure is very simple, the system is appropriate for low cost high data rate applications. However, the performance of the system is heavily affected by the selection of code set. In this paper, we investigate the bit error performance of the system according to the combination of orthogonal codes, and we propose algorithms for selecting code set.

1 Introduction

As a demand for wireless multimedia communication services is growing, the transmission schemes providing high speed and variable rate with different quality of service become necessary. Possible candidates of such systems are the variable spreading gain CDMA(VSG-CDMA) system[1,2,3] and the multi-code CDMA system[4,5]. The VSG-CDMA can adjust the spreading gain for variable rate users depending on the input traffics and control the signal power to meet the quality of services(QoS). When a user wants to transmit data at a very high bit rate in VSG-CDMA, the spreading gain may become too small to reject the interference among users.

In multi-code CDMA the transmitting signal is a linear sum of multiple orthogonal codes. The multi-code CDMA system changes the number of code channels based on the data rate of the source, hence a constant processing gain is maintained without the problem of very small processing gain of high rate users. However, the transmitting signal of the multi-code CDMA system has a large amplitude fluctuation as a result of linear sum of multi-code signals, and a highly linear power amplifier may be required for transmission. In mobile communications high power efficiency of the amplifier

may be important especially on the up-link. In order to obtain high power efficiency the power amplifier with nonlinear characteristic is used [6][7]. If the multi-code signal is amplified with power efficient amplifier, the nonlinear distortion becomes very large due to large dynamic range of the signal envelope, hence the bit error rate(BER) performance significantly degrades.

To overcome envelope fluctuation problem of multi-code CDMA a number of techniques have been proposed. Precoding scheme[8] realizes the constant amplitude by inserting precoded bits into the information bit sequence before multi-code signaling. However, the precoding scheme causes a loss in the information rate. In multi-dimensional multi-code scheme[9], the signaling is performed both in code and time spaces to reduce the information rate loss. However, as the number of channels increases the BER performance degrades because the signal energy carried on precoding channels with no information is wasted.

Pulse width CDMA(PW-CDMA)[10] is a multi-code transmission technique that performs pulse width modulation on the multi-level signal synthesized from multiple orthogonal codes. With polar signaling the signal amplitude of PW-CDMA is maintained to be constant, and the modulation and demodulation circuits become simple. However, as the number of channels increases the bandwidth increases due to the reduction of unit pulse width, and the BER performance degrades because non-optimal filter is used at the receiver. To compensate for an increase in bandwidth, pulse width modulation with polarity alternation or level clipping on the multi-code signal has been proposed[10].

In this paper, we consider an orthogonal multi-code CDMA system with binary level clipping. It is assumed that odd number of channels is used. Then the multi-code signal which is the sum of orthogonal codes has $M+1$ levels, $\{-M, -M+2, \dots, -1, 1, \dots, M\}$, where M is the number of channels. With binary level clipping, the truncated signal has constant amplitude. Due to level clipping, the BER performance degrades compared with conventional multi-code CDMA. In [11] BER performance of the system is investigated. The level clipping block in the system introduces nonlinear truncation noise. The amount of the truncation noise is dependent on the spreading codes and the information bits. In conventional orthogonal multi-code system, Walsh code is used. We may want to use other classes of orthogonal code. Possible codes include orthogonal m-sequence and orthogonal Gold code. Given a class of orthogonal code, there are number of choices of selecting M codes from the universal code set. As observed from the simulation results, choice of code set heavily affects the system performance. In this paper, we compare three classes of orthogonal code: Walsh code, orthogonal m-sequence, and orthogonal Gold code. We then propose an algorithm for finding 'good' combination of codes in the given code family.

The paper is organized as follows. Section 2 describes the binary multi-code CDMA system. In section 3 we formulate the problem and present the code selection algorithms. Section 4 presents the numerical examples and simulation results. A conclusion is given in section 5.

2 The Multi-code CDMA System with Level Clipping and PW Modulation

Fig. 1 shows the baseband system model of the orthogonal multi-code CDMA system with level clipping and pulse width modulation. Bit sequences $b_1, \dots, b_M$ with $b_m \in \{+1, -1\}$ are serial to parallel converted information bits. Information bit on each channel is spread by an orthogonal code with length of N chips.

Fig. 1. Transmitter model

During bit interval T_b data bit on each channel is multiplied by the orthogonal code with N chip length. We assume that $T_b = N T_c$ where T_c is the chip time. In the analysis given below, we consider one bit interval, i.e. $0 \leq t \leq T_b$. The sum of spread sequences produces the multi-level signal $d(n)$, which can be represented by

$$d(n) = \sum_{m=1}^M b_m c_m(n) \quad 0 \leq n \leq N-1 \quad (1)$$

where $b_m \in \{+1, -1\}$ is the information bit of the m th channel and $c_m(n)$ is the n th chip data of the spreading code for the m th channel. The spreading codes are assumed to be orthogonal:

$$\sum_{n=0}^{N-1} c_m(n) c_k(n) = N \delta_{m,k} \quad (2)$$

The number of channels, M , is assumed to be odd, which results in $M+1$ levels of multi-code signal, $d(n)$, with values $\{-M, -M+2, \dots, -1, 1, \dots, M-2, M\}$. Pulse width modulation is performed chip by chip on the multi-level signal. Pulse width of the signal during each chip time is proportional to the value of $d(n)$. If the number of channels is large, the minimum pulse width is very small, and the bandwidth of the system becomes very large. By inserting the level clipping block the amplitude of $d(n)$ is limited to a reduced number of levels, hence the bandwidth is reduced. It has been shown in [11] that level clipping in multi-code PW-CDMA system gives BER performance improvement in addition to bandwidth reduction. Fig. 2 shows an exam-

ple of waveforms of the conventional multi-code CDMA, PW-CDMA, and binary level clipped multi-code CDMA when the number of channels is 5.

Fig. 2. Signal waveforms of multi-code CDMA, PW-CDMA, Binary level clipped PW-CDMA

In this paper, we consider binary level clipping, which implies that the output of the level clipper is $\tilde{d}(n) = \pm 1$. So, the PW modulated signal of the n th chip is the antipodal rectangular pulse with pulse width T_c , i.e.

$$s_n(t) = \begin{cases} p_{T_c}(t) & \text{if } \tilde{d}(n) = 1 \\ -p_{T_c}(t) & \text{if } \tilde{d}(n) = -1 \end{cases} \quad (3)$$

and the PW modulated signal $s(t)$ may be represented by

$$s(t) = \sum_{n=0}^{N-1} s_n(t - nT_c). \quad (4)$$

Fig. 3. Receiver model

Fig. 3 shows the baseband receiver model of the system. The received signal is de-spread by the orthogonal signal and integrated to generate the decision variables as in the conventional multi-code CDMA system. The decision variable for the first channel can be expressed by

$$z_1 = \int_0^{T_b} r(t)c_1(t) dt = s_o + n_o . \quad (5)$$

where, s_o and n_o are the signal component and the noise component, respectively.

3 Selection of Code Set

3.1 Problem Formulation

The system under consideration includes level clipping block, which introduces nonlinear truncation noise. The amount of the truncation noise is dependent on the spreading codes and the information bits. In conventional orthogonal multi-code system, Walsh code is used. We may want to use other classes of orthogonal code. Possible codes include orthogonal m-sequence and orthogonal Gold code which are obtained by padding zero to the usual m-sequence and Gold codes with cross-correlation -1, respectively. Given a class of orthogonal code, there are number of choices of selecting M codes from the universal code set. As observed from the simulation results, choice of code set heavily affects the system performance. In this paper, we compare three classes of orthogonal code: Walsh code, orthogonal m-sequence, and orthogonal Gold code. We then propose an algorithm for finding ‘good’ combination of codes in the given code family.

We will describe the transmitter and receiver operations in terms of vectors and matrices. Assume the processing gain is $N = 2^k$ for some integer k . Let

$$\underline{c}_m = [c_{m1} \ c_{m2} \ \cdots \ c_{mN}] \quad (6)$$

be the m th code vector composed of the orthogonal code of length N . And let a universal set $\mathbf{CS} = \{\underline{c}_1 \ \underline{c}_2 \ \cdots \ \underline{c}_L\}$ be a collection of all code vectors associated with a given type of spreading code. L is the number of code vectors in the universal code set, and M is assumed to be less than L . For Walsh code $L=N$, and for orthogonal m-sequence $L=N-1$. Now the problem can be stated as follows. For given number of channels, M , select a subset composed of M code vectors from the universal set to provide the best BER performance.

The transmitter and the receiver models as shown in Fig. 1 and Fig. 3 can be represented in discrete form as shown in Fig. 4 and Fig. 5, respectively. Note that with binary level clipping PW signaling module can be excluded in Fig. 4. Linear combination of code vectors produces multi-level signal $\underline{d}$ which can be written as

$$\begin{aligned} \underline{d} &= [d_1 \ d_2 \ \cdots \ d_N] \\ &= \underline{b} \mathbf{A} \end{aligned} \quad (7)$$

where $\underline{\mathbf{b}}$ is an $1 \times M$ vector composed of information bits on M channels. $\mathbf{A}$ is an $M \times N$ matrix whose row is an orthogonal code vector:

$$\mathbf{A} = \begin{bmatrix} \underline{\mathbf{c}}_1 \\ \underline{\mathbf{c}}_2 \\ \vdots \\ \underline{\mathbf{c}}_M \end{bmatrix} \quad (8)$$

with

$$\underline{\mathbf{c}}_m^T \underline{\mathbf{c}}_n = \begin{cases} N & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}. \quad (9)$$

In the block diagram of transmitter model, $F[\cdot]$ indicates binary level clipping operation. Then the transmitted signal for $0 \leq t \leq T_b$ can be expressed as

$$\underline{\mathbf{s}} = [s_1 \ s_2 \ \cdots \ s_N] = F[\underline{\mathbf{d}}]. \quad (10)$$

Level clipping introduces nonlinear truncation noise. In order to analyze the truncation effect with relation code combination, we neglect the noise from the communication channel. The vector of decision variables can be written as

$$\begin{aligned} \underline{\mathbf{z}} &= [z_1 \ z_2 \ \cdots \ z_M] \\ &= \underline{\mathbf{s}} \mathbf{A}^T \\ &= F[\underline{\mathbf{b}} \mathbf{A}] \mathbf{A}^T \end{aligned} \quad (11)$$

Fig. 4. Transmitter model

Fig. 5. Receiver model

3.2 Code Selection Algorithm

As mentioned earlier, level clipping introduces nonlinear truncation. The clipping effects appear as magnitude reduction or sign conversion of decision variables. If magnitude of decision variable is reduced the output SNR decreases and the system becomes vulnerable to channel noise. If the sign of decision variable is changed to be opposite to the sign of information bit, there may be irreducible error, i.e. the BER performance curve exhibits irreducible floor.

For a given type of spreading code, suppose there are L codes in the universal code set. If there are M channels with $M < L$, then there are ${}_L C_M$ possible code combinations, where

$${}_n C_r = \frac{n!}{r!(n-r)!} . \quad (12)$$

The truncation noise is a function of code set and information bits. If the system has M channels and information bits are binary, i.e. $b_m \in \{+1, -1\}$, $m = 1, \dots, M$, there are 2^M possible information bit patterns input to the system. Then there are $2^M \times M$ corresponding decision variables. Our goal is to find the best code combination which leads to the best BER performance. The code selection algorithm traces the ${}_L C_M$ combinations of codes and investigates the statistical behavior of the decision variables to the input patterns. We suggest four algorithms. Algorithm A and B are based on the average distance of the decision variables. Algorithm C and D are based on the histogram of decision variables.

Before presenting the algorithms we define variables necessary to describe the algorithms. For the system with M channel input, there are 2^M possible M -tuple input

vectors. Let k be the index for those input vectors, i.e. $k \in \{1, \dots, 2^M\}$. For example, $\underline{b}(1) = [1, \dots, 1]$, $\underline{b}(2) = [1, \dots, 1, -1]$, $\underline{b}(2^M) = [-1, \dots, -1]$. Let

$$\underline{z}(k) = [z_1(k), z_2(k), \dots, z_M(k)] \quad (13)$$

be the vector of decision vector for the k th input pattern. Define

$$\underline{z}^{inv}(k) = \left[\frac{1}{z_1(k)}, \frac{1}{z_2(k)}, \dots, \frac{1}{z_M(k)} \right]. \quad (14)$$

as a vector obtained by inverting each element in $\underline{z}(k)$. We then define variables regarding the distribution of decision variables $\{z_m(k)\}$ for all m and k , which are required to describe algorithm C and D. Let $\{v(n), n=1, 2, \dots\}$ be the values of $\{z_m(k)\}$ and $v(n)$'s are sorted such that $v(1) < v(2) < \dots$. Define the number of decision variables whose value is $v(n)$, i.e.

$$N_n = \text{histogram}(v(n)) \quad n = 1, 2, \dots \quad (15)$$

The algorithms of code selection are stated as follows.

Common part

Traces the ${}_L C_M$ combinations of codes.

Discard the code set if $\text{sign}(z_m(k)) \neq \text{sign}(b_m(k))$ for any m, k .

Algorithm A

Choose the code set that minimizes

$$\eta = E_k \left\{ \max_m \frac{1}{z_m(k)} \right\} = E_k \left\{ \left\| \underline{z}^{inv}(k) \right\|_\infty \right\}. \quad (16)$$

Algorithm B

Choose the code set that minimizes

$$\eta = E_k \left\{ \left\| \underline{z}^{inv}(k) \right\|_q \right\} \quad (q = 2). \quad (17)$$

Algorithm C

Choose the code set that minimizes

$$\eta = \frac{1}{v(1)}. \quad (18)$$

Algorithm D

Choose the code set that minimizes

$$\eta = \frac{N_1}{\{v(1)\}^q} + \frac{N_2}{\{v(2)\}^q} + \dots \quad (q = 3). \quad (19)$$

4 Numerical Examples and Discussion

We have analyzed the statistical behavior of decision variables to observe the effects of level clipping as a function of code combination. We have evaluated the code set selection algorithms proposed in section 3 by comparing the results with those from simulations. As spreading codes, we consider three types of codes: Walsh code, an orthogonal m-sequence, and an orthogonal Gold code.

Example 1: $N=32, M=7$

Though there are $M \times 2^M$ decision variables for all possible input vectors, the distribution characteristics are one of three types as shown in Fig. 6. For each distribution pattern there is a corresponding code set.

Fig. 6. Distribution patterns of decision variables

Table 1 shows the figure of merit given by each algorithm which indicates how well the code set will perform. The last column in the table indicates BER performance from simulation for given code set. The results indicate that algorithm B and D work well. Fig. 7 shows the simulation results for three code sets. The dotted line is the BER performance when a bad code set is chosen. The code selection algorithms exclude this code set. It is observed that there is irreducible BER floor for bad selection of codes.

Table 1. Comparison of code selection algorithms ($N=32, M=7$)

	A		B		C		D		simulation
	η	rank	η	rank	η	rank	η	rank	rank
I	.125	1	.281	1	.125	1	1.134	1	1
II	.125	1	.298	2	.125	1	1.367	2	2
III	.125	1	.311	3	.125	1	1.529	3	3

Fig. 7. BER performance for given code sets ($N=32$, $M=7$)

For three classes of spreading codes, Walsh code, orthogonal m-sequence, and orthogonal Gold code, we have investigated the statistics of the decision variables. We have observed the same distribution as given in Fig. 6. This result indicates that these three orthogonal codes provide the same BER performance if code combination is properly selected. Fig. 8 shows the BER performance for three classes of codes with the best code combination suggested by the algorithms. As expected, the same simulation results are obtained.

Fig. 8. Simulation results for three classes of codes

Example 2: $N=64, M=9$

We have carried out the same procedure as in example 1. In this example, we have observed six major distribution types in histogram of decision variables. Table 2 shows the figure of merit given by each algorithm which indicates how well the code set will perform. The results indicate that algorithm D works well in the sense that the algorithm expects almost the same result with that from the simulations. Fig. 9 shows the simulation results for three code sets.

Table 2. Comparison of code selection algorithms ($N=64, M=9$)

type	A		B		C		D		Simulation
	η	rank	η	rank	η	rank	η	rank	rank
I	.0647	1	.1791	1	.0714	1	1.0151	1	1
II	.0647	1	.1796	2	.0714	1	1.0278	2	2
III	.0651	3	.1847	4	.0833	3	1.1333	3	3
IV	.0764	4	.1836	4	.0833	3	1.1522	4	4
VI	.0875	5	.1855	5	.1	5	1.2002	5	5
VII	.1198	7	.2070	7	.125	6	1.9150	7	6
VIII	.1120	6	.2059	6	.125	6	1.8970	6	7

Fig. 9. BER performance for given code sets ($N=64, M=9$)

5 Conclusion

Multi-code PW-CDMA is a transmission technique that linear sum of orthogonal codes is level clipped and pulse width modulated. The system has the capability of multi-code CDMA system of supporting multimedia traffics without difficulty of handling high rate traffic in VSG-CDMA. In addition, the system has the feature of constant envelope transmission. Hence, the system is appropriate for low cost high data rate applications. We consider the multi-code PW-CDMA system with binary level clipping. The performance of the system is heavily affected by the selection of code set. In this paper, we investigate the bit error performance of the system according to the combination of orthogonal codes, and we propose algorithms for selecting code set. We also compare three classes of spreading code: Walsh code, orthogonal m-sequence, and orthogonal Gold code. From the analysis it was observed that these three orthogonal codes provide the same BER performance if code combination is properly selected.

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