DECOMPOSITION OF FLUORESCENT ILLUMINANT SPECTRA FOR ACCURATE COLORIMETRY

G. Sharma and H. J. Trussell

Electrical & Computer Engineering Dept.
North Carolina State University, Raleigh, NC 27695-7911

ABSTRACT

Fluorescent lamps are widely used in the office environment and in common desktop scanners. These lamps are characterized by the nearly monochromatic emission lines present in their radiant spectrum. Due to the presence of spectral lines special care is required in the computation of tristimulus values under these illuminants. Accurate computation of the tristimulus values can be performed digitally by modeling the radiance as the sum of a continuous spectrum emitted by the phosphors and weighted impulses corresponding to the emission lines[2, 14, 3]. This paper considers a scheme for estimating the continuous spectrum and the strengths of the impulses from measurements of the spectrum with band limited sensors. The impact of noise in measured data on the accuracy of the final color computation is also considered.

1. INTRODUCTION

Color is normally specified in terms of tristimulus values. The CIE XYZ tristimulus values for a reflectance $r(\lambda)$ under a viewing illuminant $l(\lambda)$ are given by

$$t_i = \int_{-\infty}^{\infty} r(\lambda)l(\lambda)m_i(\lambda)d\lambda \quad i = 1, 2, 3 \qquad (1)$$

where $\{m_i(\lambda),\ i = 1, 2, 3\}$ are the CIE XYZ color matching functions[1] [11].

The integral in Eqn. (1) is usually approximated by a summation and tristimulus values are computed digitally by multiplying sampled spectra with CIE color matching functions over the visible spectrum and summing the result. For band limited spectra the sampling interval is important in determining the accuracy of this computation[7]. Fluorescent lamp spectra are not band limited and thus present a special problem. It has been shown that [2, 14] color values under a fluorescent illuminant, $l(\lambda)$, can be computed more accurately by using a decomposition of the spectrum

$$l(\lambda) = c(\lambda) + \sum_{k=1}^{q} \alpha_k \delta(\lambda - \lambda_k) \qquad (2)$$

where $c(\lambda)$ is the smooth and continuous (band limited) spectrum emitted by the phosphors, $\{\lambda_1, \lambda_2 \ldots \lambda_q\}$ are the locations of the q spectral peaks and α_k is the strength of the spectral line at wavelength λ_k emitted by the vapor in the lamp (typically mercury).

The smooth spectrum can be represented by its samples at 10 nm increments thus yielding a representation of the whole spectrum in terms of these samples along with the strengths $\{\alpha_k\}_{k=1}^{q}$ and the locations $\{\lambda_k\}_{k=1}^{q}$ of the spectral peaks. The purpose of this research is to estimate the parameters of the above model based on measurements of the spectral radiance.

2. ESTIMATION OF MODEL PARAMETERS

The measurement process can be modeled as yielding samples of

$$\begin{aligned} y(\lambda) &= l(\lambda) * h(\lambda) + n(\lambda) \\ &= c(\lambda) * h(\lambda) + \sum_{k=1}^{q} \alpha_k\, h(\lambda - \lambda_k) + n(\lambda) \\ &= g(\lambda) + \sum_{k=1}^{q} \alpha_k\, h(\lambda - \lambda_k) + n(\lambda) \qquad (3) \end{aligned}$$

where $h(\lambda)$ is the instrument impulse response, $n(\lambda)$ is the measurement noise, $*$ denotes the convolution operation and $g(\lambda) = c(\lambda) * h(\lambda)$. The instrument impulse response $h(\lambda)$ is typically low pass in nature with a passband that ensures the Nyquist criterion is met by the samples.

The problem of estimating $l(\lambda)$ from $y(\lambda)$ is an ill-posed problem. However, *a priori* information about the nature of the different functions and their bandwidths can be used to formulate the estimation as a signal restoration problem. The instrument impulse response is analogous to an optical aperture and will be a smooth function with limited support in wavelength. It is also known *a priori* that $c(\lambda)$ can be sampled at 10 nm increments[7]. Hence $c(\lambda)$ and consequently $g(\lambda) = c(\lambda) * h(\lambda)$ are band limited to $1/20$ $cycles/nm$.

The peculiar nature of the signal due to the mix of continuous and discrete components does not allow a direct formulation of the problem as a one step restoration even if the instrument impulse response $h(\lambda)$ is known. A minimum mean squared error (MMSE) estimation scheme would not only require additional knowledge of signal and noise statistics but would also require an extremely fine sampling to achieve the desired resolution for the spectral peaks. The consequent increase in the dimensionality of the problem would make it a highly underdetermined estimation problem. An alternative would be to use the method of projections onto convex sets(POCS). The set of signals having the structure of Eqn. 2 is a convex set but the problem cannot be cast as a single step restoration using POCS since the set does not lie in the Hilbert space of square integrable functions on account of the continuous delta functions. In spite of these limitations the following multistep procedure can be used to estimate the model parameters:

1. Use limited "spatial" extent of $h(\lambda)$ to obtain an estimate

   $$\hat{g}(\lambda) = y(\lambda) \approx g(\lambda) \qquad \forall \lambda \notin \cup_{i=1}^{q} (\lambda_i - \Lambda, \lambda_i + \Lambda)$$

   where $h(\lambda)$ is assumed to be close to zero outside the interval $(-\Lambda, \Lambda)$.

2. Use band limited extrapolation [8] to estimate $\hat{g}(\lambda) \approx g(\lambda)$ for $\lambda \in \cup_{i=1}^{q} (\lambda_i - \Lambda, \lambda_i + \Lambda)$

3. Use estimate, $\hat{g}(\lambda)$ of $c(\lambda) * h(\lambda)$ to obtain an estimate of $u(\lambda) = \sum_{k=1}^{q} \alpha_k \, h(\lambda - \lambda_k)$

   $$\hat{u}(\lambda) = y(\lambda) - \hat{g}(\lambda) \approx u(\lambda)$$

4. Model $h(\lambda)$ as a zero mean Gaussian[1] and estimate the variance $\hat{\sigma}$ and amplitude parameters ($\hat{\alpha_k}$'s) from $\hat{u}(\lambda) \approx \sum_{k=1}^{q} \alpha_k \, h(\lambda - \lambda_k)$ using nonlinear least squares. Thus obtain an estimate for $h(\lambda)$,viz., $\hat{h}(\lambda) = \frac{1}{\sqrt{2\,\pi}\,\hat{\sigma}} \exp(-\frac{\lambda^2}{2\,\hat{\sigma}^2})$,

5. Deconvolve $\hat{g}(\lambda) \approx c(\lambda) * h(\lambda)$ using $\hat{h}(\lambda)$ to obtain estimate $\hat{c}(\lambda) \approx c(\lambda)$.

[1]In [10] a triangular instrument impulse response is assumed for restoration of smooth spectra sampled at rates much lower than the 2nm or less sampling used in accurate spectraradiometers. Trials with measured data at 2 nm are better matched with a Gaussian.

The "spatial" extent Λ of the instrument impulse response can be estimated conservatively by a visual examination of the measured spectra. This step is unnecessary if one can make test measurements with a clear mercury lamp in which the continuous background due to the phosphors is absent. It is noted that clear mercury lamps are commercially available. Such an experiment will also yield better estimates of the impulse response $h(\lambda)$.

In its raw form the nonlinear least squares problem of step 4 is multidimensional with a large number of local minima. However, using the well known solution to the linear least squares problem, it is readily reduced to a minimization involving the variance, $\hat{\sigma}$, alone. Once the optimal variance has been determined the optimal amplitude parameters are immediately known from the linear least squares solution.

Fig. 1 shows the radiant spectrum of a "standard daylight" mercury vapor fluorescent lamp measured using a spectroradiometer with a 2nm sampling interval. Fig. 2 shows the decomposition of the same illuminant obtained by the above procedure. For this experiment Λ was set to $10nm$ and the bandwidth for the band limited extrapolation in step 2 was computed from the fact that a $10nm$ sampling rate is sufficient for representing $g(\lambda)$. For the spectral peak locations $\{\lambda_1, \lambda_2 \ldots \lambda_q\}$ the known locations of spectral emission lines of mercury vapor at 404.7nm, 435.8nm and 546.1nm and at 577.8nm[9, pp. 213] [2] were used. A reconstruction of $y(\lambda)$ formed from the estimated decomposition using the estimated $h(\lambda)$ is indistinguishable from the original $y(\lambda)$ on the scale of the graph.

3. ESTIMATION ACCURACY AND EFFECT OF NOISE

For investigating the performance of the estimation scheme in the presence of noise the measurement process was simulated using a "standard daylight" mercury vapor fluorescent lamp spectrum given in [11]. The tabulated radiance data was in the form of 10 nm samples of the continuous spectrum emitted by the phosphors and strengths of the emission lines of Mercury in the visible region. To simulate the measurement process the continuous part was interpolated to 2 nm and digitally convolved with a Gaussian to get the measured data. White Gaussian noise at the desired signal to noise ratio (SNR) was added to the measured data and the resulting samples were used in the estimation scheme. The procedure was repeated for different SNR values.

[2]The line at 577.8nm is actually a doublet at 577.0 and 579.0nm

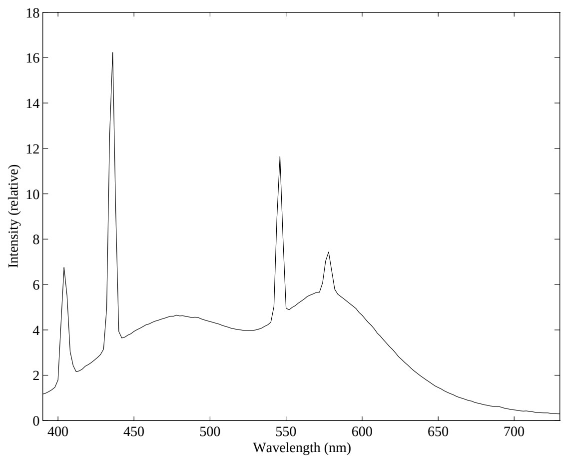

Figure 1: Fluorescent Illuminant Spectrum measured at 2 nm.

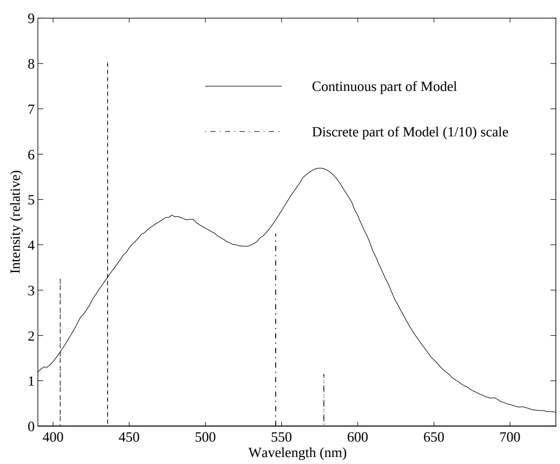

Figure 2: Decomposition of Spectrum in Fig. 1

The band limited extrapolation of steps 1 and 2 performs poorly if the available data is noisy. A restoration/extrapolation scheme using the method of projections onto convex sets (POCS) [4, 5] was therefore used instead of the band limited extrapolation in steps 1 and 2. Known constraints on the signal were used to set up the projections onto convex sets (positivity, band limitedness, noise variance, outliers [6, 13]).

Due to the incomplete nature of the information available it is not possible to estimate the decomposition exactly even in the absence of noise. Noise in the measured spectrum will further degrade the accuracy of the decomposition.

Fig. 3 and Fig. 4 show the results of the simulations at 40dB and 30dB respectively. The plots show the measurements along with the estimated decomposition for comparison.

While the estimation error can be measured by the usual metrics of signal restoration such as mean squared error (MSE) or maximum deviation, the metrics are not very meaningful in the present context due to the presence of the distinct continuous and discrete parts with errors that are ameliorated in the final color computation. For instance, if during the band limited extrapolation one overestimates $g(\lambda)$ around a spectral peak the band limited part, $\hat{c}(\lambda)$, is higher in the neighborhood of the peak but the corresponding value of the peak strength is lower. When the decomposition is used to estimate tristimulus values with reflective surfaces having smooth (band limited) reflectance spectra these errors will act in opposite directions reducing the total error.

The true indicator of the decomposition accuracy is the error in color perception. The error in the estimates leads to erroneous tristimulus values which in turn lead to errors in perceived color. The MSE in the CIE XYZ tristimulus space and in the perceptually uniform Lab space (called the $\triangle E$ error) are therefore considered for evaluating the performance of the estimation method.

In terms of the fluorescent illuminant model the calculation of tristimulus values in eqn. (1) can be rewritten as

$$\begin{aligned} t_i &= \int_{-\infty}^{\infty} r(\lambda)c(\lambda)m_i(\lambda)d\lambda + \sum_{k=1}^{q} \alpha_k \, m_i(\lambda_k)\, r(\lambda_k) \\ &\approx \sum_{n=0}^{N} m_i(\lambda_0 + n\triangle\lambda)\, \hat{c}(\lambda_0 + n\triangle\lambda)\, r(\lambda_0 + n\triangle\lambda)\triangle\lambda \\ &\quad + \sum_{k=1}^{q} \hat{\alpha}_k \, m_i(\lambda_k)\, r(\lambda_k) \end{aligned} \tag{4}$$

where $\{\hat{c}(\lambda_0 + n\triangle\lambda)\}_{n=0}^{N}$ are the estimated samples of $c(\lambda)$ (the continuous part of $l(\lambda)$) and $\hat{\alpha}_k$ is the esti-

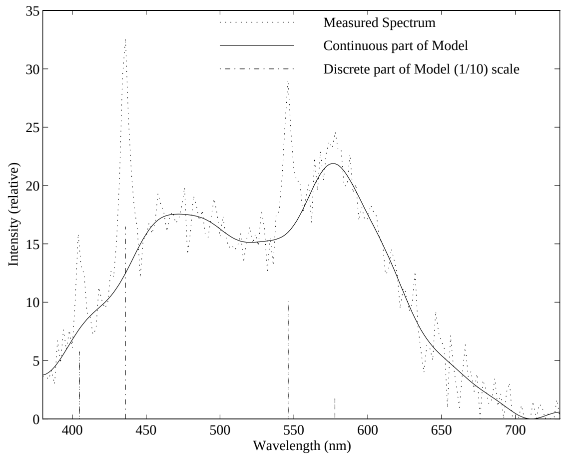

Figure 3: Decomposition of Fluorescent Illuminant Spectrum at SNR of 40dB.

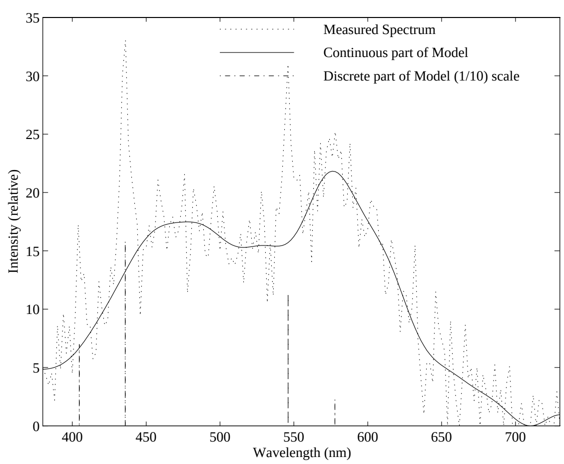

Figure 4: Decomposition of Fluorescent Illuminant Spectrum at SNR of 30dB.

Table 1: MSE in CIE XYZ space for the model and for direct 2nm computation

	MSE (dB)					
	Model			2nm Comp.		
SNR (dB)	X	Y	Z	X	Y	Z
50	-73.58	-69.11	-58.76	-56.12	-56.93	-38.72
45	-72.25	-67.02	-58.72	-56.28	-57.34	-38.74
40	-70.66	-64.63	-58.72	-56.48	-57.89	-38.76
35	-68.10	-61.29	-58.75	-56.75	-58.66	-38.78
30	-64.86	-57.31	-58.64	-57.11	-59.73	-38.82

Table 2: Average $\triangle E$ errors for the model and for direct 2nm computation

	Avg. $\triangle E$ *error*				
SNR (dB)	50	45	40	35	30
Model	0.937	0.964	1.022	1.161	1.457
2nm Comp.	3.219	3.240	3.269	3.311	3.373

mated strength of the peak at λ_k, $(k = 1, 2, \ldots q)$.

For numerically quantifying the performance of this estimation scheme tristimulus values were computed by Eqn. 4 using the true values of the parameters for a set of 64 reflectances from the Munsell chip set. The computation of tristimuli was repeated using the estimated parameters in Eqn. 4 and directly with the 2 nm "measured" data. These were used to obtain the mean squared errors in CIE XYZ space. The CIE XYZ data was then converted to Lab and CIE Lab $\triangle E$ errors were computed for each chip and the average $\triangle E$ error and maximum $\triangle E$ error was computed for both cases. The procedure was repeated for different SNR's. The mean squared tristimulus errors are tabulated in Table 1; Table 2 and Table 3 give the average and maximum $\triangle E$ errors respectively. The table indicates that the model performs extremely well with most of the errors in color perception being below the visual threshold of 2 dB for the entire SNR range considered. Its performance is much better than that of the 2 nm computation using the measured data directly.

4. CONCLUSIONS

A model for fluorescent spectra based on the physics of these lamps was presented and scheme for estimating the model parameters from measured data was developed. The model yields a parsimonious representation of the fluorescent illuminant that allows accurate

Table 3: Maximum $\triangle E$ errors for the model and for direct 2nm computation

	Max. $\triangle E$ *error*				
SNR (dB)	50	45	40	35	30
Model	1.614	1.839	2.130	2.588	3.271
2nm Comp.	6.003	6.090	6.207	6.364	6.575

computation of tristimulus values. The proposed estimation scheme exploits the *a priori* knowledge of signal structure and bandwidths to estimate the model parameters. Numerical results in terms of $\triangle E$ errors obtained by using the model for tristimulus value computation indicate its value in obtaining accurate color measurements.

5. REFERENCES

[1] *Colorimetry*; 2nd Ed., CIE Publication 15.2, Central Bureau of the CIE, Paris, 1986.

[2] H. J. Trussell and M. Kulkarni, "Estimation of Color Under Fluorescent Illuminants", IEEE ICIP-94.

[3] W. H. Venable, "Accurate Tristimulus Values from Spectral Data," *Color Res. Appl.*, Vol. 14, No. 5, Oct. 1989.

[4] L.M. Bregman, "The Method of Successive Projection for finding a Common point of Convex Sets," *Dokl. Akad. Nauk. USSR*, Vol. 162, No.3,1965,pp. 688-692.

[5] L.G. Gubin, B.T. Polyak and E.T. Raik, "The Method of Projections for finding the Common point of Convex Sets," *USSR Comput. Math. and Phys.*, Vol. 7,No. 6,1967,pp. 1-24.

[6] H. J. Trussell and M. R. Civanlar, "Feasible Solution in Signal Restoration", *IEEE Trans. ASSP*, Vol. ASSP-32, No. 2, pp. 201-212, April 1984.

[7] H. J. Trussell and M. S. Kulkarni, "Sampling and Processing of Color Signals", submitted to IEEE Trans. Image Processing.

[8] J. L. C. Sanz and T. S. Huang, "Support-Limited Signal and Image Extrapolation", Chapter 1, *Advances in Computer Vision and Image Processing, A Research Annual, Image Reconstruction From Incomplete Observations*, T. S. Huang, Ed., JAI Press, 1984.

[9] John F. Waymouth, *Electric Discharge Lamps*, M.I.T. Press, 1971.

[10] E. I. Stearns and R. E. Stearns, "An Example of a Method for Correction Radiance Data for Band-pass Error," *Color Res. Appl.*, Vol. 13, No. 4, pp. 257-259, Aug. 1988.

[11] G. Wyszecki and W. S. Styles, *Color Science: Concepts and Methods, Quantitative Data and Formulae*, John Wiley & Sons, Inc., 1967.

[12] A. E. Savakis and H. J. Trussell, "On the accuracy of PSF representation in image restoration", *IEEE Trans. Image Processing*, Vol. 2, No. 2, pp. 252-259, April 1993.

[13] H. J. Trussell, "Application of Set Theoretic Models to Color Systems", *Color Res. Appl.*, Vol. 16, No. 1, February 1991.

[14] MacAdam, David L., *Color Measurement: Theme and Variations*, Springer-Verlag, 1981.