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Image Compression using Coding of Wavelet Coefficients – A Survey

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Abstract
Due to the increasing traffic caused by multimedia information and digitized form of representation of images; image compression has become a necessity. New algorithms for image compression based on wavelets have been developed. These methods have resulted in practical advances such as: superior low-bit rate performance, continuous–tone and bit-level compression, lossless and lossy compression, progressive transmission by pixel, accuracy and resolution, region of interest coding and others. We concentrate on the following methods of coding of wavelet coefficients, in this paper: EZW (embedded zero tree wavelet) algorithm, SPIHT (set partitioning in hierarchical trees) algorithm, SPECK (Set Partitioned Embedded Block Coder), WDR (wavelet difference reduction) algorithm, and ASWDR (adaptively scanned wavelet difference reduction) algorithm. These are relatively recent algorithms which achieve some of the lowest errors per compression rate and highest perceptual quality

Keywords: *Image compression, PEG, Wavelets, EZW, SPIHT, SPECK, EBCOT, WDR, ASWDR, SFQ, CREW, EPWIC, SR, SFQ*

1. Introduction

Uncompressed multimedia (graphics, audio, Video) data requires considerable storage capacity and transmission bandwidth despite rapid progress in mass storage density, processor speeds and digital communication system performance, demand for data storage capacity and data transmission bandwidth continues to outstrip the capabilities of available technologies. The recent growth of data intensive multimedia-based web applications have not only the need for more efficient ways to encode signals and images but have made compression of such signals central to storage and communication technology. The Table.1 shows Multimedia data types and uncompressed storage space, transmission bandwidth and transmission time required. The above information clearly illustrates the need for sufficient storage space, large transmission bandwidth and long transmission time for image, audio and video data. At the present state technology, the only solution is to compress multimedia data before its storage and transmission, and decompress it at the receiver for playback. For example for a compression Ratio of 32:1 ,the space, bandwidth and the transmission time requirements can be reduced by a factor of 32,with acceptable quality.

1.1 Principles behind Compression

A Common characteristic of most images is that the neighboring pixels are correlated and therefore contain redundant information. The foremost task then is to find less correlated representation of the Image. Two fundamental components of compression are redundancy and irrelevancy reduction. Redundancy reduction aims at removing duplication from the signal source(image/video).Irrelevancy reduction omits parts of the signal that will not be noticed by the signal receiver namely Human Visual System(HVS).In general three types of redundancy can be identified

- Spatial Redundancy or correlation between neighboring pixel.
- Spectral redundancy or correlation between different color planes or spectral bands
- Temporal redundancy or correlation between adjacent frames in a sequence of images (in video applications).

Image compression research aims at reducing the number of bits needed to represent an image by removing the spatial and spectral redundancies as much as possible. In many different fields, digitized images are replacing conventional analog images as photograph or x-rays. The volume of data required to describe such images greatly slow transmission and makes storage prohibitively costly. The information contained in images must, therefore, be compressed by extracting only visible elements, which

Multimedia Data	Size/duration	Bits/pixel (or) Bits/sample	Uncom pressed size (B-bytes)	Trans -mission Bandwidth (b-bits)	Trans Mission Time(28.8K Modem)
Page of text	11" X 8.5"	Varying resolution	4-8 KB	32-64 Kb/page	1.1-2.2 Secs
Telephone Quality speech	10 Secs	8 bps	80 KB	64 Kb/Sec	22.2 Secs
Grayscale image	512 X 512	8 bpp	262 KB	2.1 Mb/image	1min 13 Secs
Color Image	512 X 512	24 bpp	786 KB	6.29 Mb/image	3min 39Secs.
Medical image	2048 X 2048	12 bpp	5.16 MB	41.3 Mb/image	23min 54Secs

Table.1 Multimedia Data

are then encoded. The quantity of data involved is thus reduced substantially. The fundamental goal of image compression is to reduce the bit rate for transmission or storage while maintaining an acceptable fidelity or image quality.

One of the most successful applications of wavelet methods is transform-based image compression (also called coding). The overlapping nature of the wavelet transform alleviates blocking artifacts, while the multiresolution character of the wavelet decomposition leads to superior energy compaction and perceptual quality of the decompressed image. Furthermore, the multiresolution transform domain means that wavelet compression methods degrade much more gracefully than block-DCT methods as the compression ratio increases. Since a wavelet basis consists of functions with both short support (for high frequencies) and long support (for low frequencies), large smooth areas of an image may be represented with very few bits, and detail added where it is needed [27].

Wavelet-based coding [27] provides substantial improvements in picture quality at higher compression ratios. Over the past few years, a variety of powerful and sophisticated wavelet-based schemes for image compression, as discussed later, have been developed and implemented. Because of the many advantages, wavelet-based compression algorithms are the suitable candidates for the new JPEG-2000 standard [34].

Such a coder operates by transforming the data to remove redundancy, then quantizing the transform coefficients (a lossy step), and finally entropy coding the quantizer output. The loss of information is introduced by the quantization stage which intentionally rejects less relevant parts of the image information. Because of their superior energy compaction properties and correspondence with the human visual system, wavelet compression methods have produced superior objective and subjective results [4].

With wavelets, a compression rate of up to 1:300 is achievable [22]. Wavelet compression allows the integration of various compression techniques into one algorithm. With lossless compression, the original image is recovered exactly after decompression. Unfortunately, with images of natural scenes, it is rarely possible to obtain error-free compression at a rate beyond 2:1 [22]. Much higher compression ratios can be obtained if some error, which is usually difficult to perceive, is allowed between the decompressed image and the original image. This is lossy compression. In many cases, it is not necessary or even desirable that there be error-free reproduction of the original image. In such a case, the small amount of error introduced by lossy compression may be acceptable. Lossy compression is also acceptable in fast transmission of still images over the Internet [22].

Over the past few years, a variety of novel and sophisticated wavelet-based image coding schemes have been developed. These include Embedded Zero tree Wavelet (EZW) [13], Set-Partitioning in Hierarchical Trees (SPIHT) [1], Set Partitioned Embedded bloCK coder (SPECK) [2], Wavelet Difference Reduction (WDR)[28], Adaptively Scanned Wavelet Difference Reduction (ASWDR) [29] , Space –Frequency Quantization (SFQ) [42], Compression with Reversible Embedded Wavelet (CREW) [3], Embedded Predictive Wavelet Image Coder (EPWIC) [5], Embedded Block Coding with Optimized Truncation (EBCOT) [25], and Stack- Run (SR) [26]. This list is by no means exhaustive and many more such innovative techniques are being developed. A few of these algorithms are briefly discussed here.

In Section 2, the preliminaries about wavelet transform of an image are discussed along with the performance metrics. The Section 3 gives an outline of the common features of the various wavelet based coding schemes. The salient and unique features of these schemes are summarized in Section 4.The Section 5 discusses about the unique features and the demerits of various coding schemes. The Section 6 concludes the paper.

2. Preliminaries

Most natural images have smooth colour variations, with the fine details being represented as sharp edges in between the smooth variations. Technically, the smooth variations in colour can be termed as low frequency variations and the sharp variations as high frequency variations. The low frequency components (smooth variations) constitute the base of an image, and the high frequency components (the edges which give the detail) add upon them to refine the image, thereby giving a detailed image. Hence, the smooth variations are demanding more importance than the details.

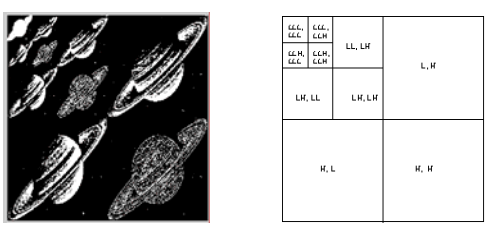
Separating the smooth variations and details of the image can be done in many ways. One such way is the decomposition of the image using a Discrete Wavelet Transform (DWT) [15], [23], [27].

Wavelets [23], [27] are being used in a number of different applications. The practical implementation of wavelet compression schemes is very similar to that of subband coding schemes [22], [21], [8]. As in the case of subband coding, the signal is decomposed using filter banks. In a discrete wavelet transform, an image can be analyzed by passing it through an analysis filter bank followed by a decimation operation. This analysis filter bank, which consists of a low pass and a high pass filter at each decomposition stage, is commonly used in image compression.

When a signal passes through these filters, it is split into two bands. The low pass filter, which corresponds to an averaging operation, extracts the coarse information of the signal. The high pass filter, which corresponds to a differencing operation, extracts the detail information of the signal. The output of the filtering operations is then decimated by two.

A two-dimensional transform can be accomplished by performing two separate one-dimensional transforms. First, the image is filtered along the x-dimension using low pass and high pass analysis filters and decimated by two. Low pass filtered coefficients are stored on the left part of the matrix and high pass filtered on the right. Because of decimation, the total size of the transformed image is same as the original image. Then, it is followed by filtering the sub-image along the y-dimension and decimated by two. Finally, the image has been split into four bands denoted by LL, HL, LH, and HH, after one level of decomposition. The LL band is again subject to the same procedure. This process of filtering the image is called pyramidal decomposition of image [22].This is depicted in Fig. 1.

The reconstruction of the image can be carried out by reversing the above procedure and it is repeated until the image is fully reconstructed.



(a) (b)

Fig.1 (a) Three level octave-band decomposition of Saturn image, and (b) Spectral decomposition and ordering.

2.1 Quantization

Quantization [22], [21] refers to the process of approximating the continuous set of values in the image data with a finite, preferably small, set of values. The input to a quantizer is the original data and the output is always one among a finite number of levels. The quantizer is a function whose set of output values are discrete and usually finite. Obviously, this is a process of approximation and a good quantizer is one which represents the original signal with minimum loss or distortion.

There are two types of quantization: scalar quantization and vector quantization. In scalar quantization, each input symbol is treated in producing the output while in vector quantization the input symbols are clubbed together in groups called vectors, and processed to give the output. This clubbing of data and treating them as a single unit, increases the optimality of the vector quantizer, but at the cost of increased computational complexity [21].

2.2 Error Metrics

Two of the error metrics used to compare the various image compression techniques are the Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR). The MSE is the cumulative squared error between the compressed and the original image, whereas PSNR is a measure of the peak error. The mathematical formulae for the two are

Error E = *Original image – Reconstructed image*

MSE = E / (SIZE OF IMAGE)

$$PSNR = 20 * \log_{10}\left(\frac{255}{\sqrt{MSE}}\right) \quad (1)$$

A lower value for MSE means lesser error, and as seen from the inverse relation between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of Signal to Noise is higher. Here, the 'signal' is the original image, and the 'noise' is the error in reconstruction. So, a compression scheme having a lower MSE (and a high PSNR), can be recognized as a better one.

Wavelet-based coding is more robust under transmission and decoding errors, and also facilitates progressive transmission of images [35]. In addition, they are better matched to the Human Visual System (HVS) characteristics. Because of their inherent multiresolution nature [15], wavelet coding schemes are especially suitable for applications where *scalability* and *tolerable degradation* are important

3. Coding Schemes for Wavelet Coefficients

Image coding utilizing scalar quantization on hierarchical structures of transformed images has been a very effective and computationally simple technique. Shapiro was the first to introduce such a technique with his EZW [13] algorithm. Different variants of this technique have appeared in the literatures which provide an improvement over the initial work. Said & Pearlman [1] successively improved the EZW algorithm by extending this coding scheme, and succeeded in presenting a different implementation based on a set-partitioning sorting algorithm. This new coding scheme, called the SPIHT [1], provided an even better performance than the improved version of EZW.

The common features for the wavelet based schemes are briefly discussed here.

All of these scalar quantized schemes employ some kind of significance testing of sets or groups of pixels, in which the set is tested to determine whether the maximum magnitude in it is above a certain threshold.

The results of these significance tests determine the path taken by the coder to code the source samples. These significance testing schemes are based on some very simple principles which allow them to exhibit excellent performance. Among these principles is the partial ordering of magnitude coefficients with a set-partitioning sorting algorithm, bit plane transmission in decreasing bit plane order, and exploitation of self-similarity across different scales of an image wavelet transform. After wavelet transforming an image we can represent it using trees because of the sub-sampling that is performed in the transform. A coefficient in a low subband can be thought of as having four descendants in the next higher subband. The four descendants each also have four descendants in the next higher subband and we see a *quad-tree* [13]

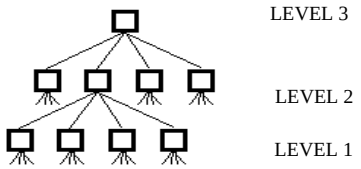


Fig 2. Quad Tree structure

emerge: every root has four leafs. Fig. 2 shows the quad tree structure. A very direct approach is to simply transmit the values of the coefficients in decreasing order, but this is not very efficient. This way a lot of bits are spent on the coefficient values. A better approach is to use a threshold and only signal to the decoder if the values are larger or smaller than the threshold. If we adopt bit-plane coding [13], [2] then our initial threshold t0 will be :

$$t_0 = 2^{\lfloor \log_2(\mathrm{MAX}(|\gamma(x,y)|)) \rfloor} \qquad (2)$$

Here MAX(.) means the maximum coefficient value in the image and $\gamma(x,y)$ denotes the coefficient. If the threshold is also transmitted to the decoder, it can reconstruct already quite a lot.

An interesting thing to note in these schemes is that all of them have relatively low computational complexity, considering the fact that their performance is comparable to the best-known image coding algorithms [1]. This feature seems in conflict with the well-known tenets of information theory that the computational complexity of a stationary source (i.e., source sample aggregates) increases as the coding efficiency of the source increases [1]. These coding schemes seem to have provided a breathing space in the world of simultaneously increasing efficiency and computational complexity.

An important characteristic that this class of coders possesses is the property of progressive transmission and embedded nature.

Progressive transmission [1] refers to the transmission of information in decreasing order of its information content. In other words, the coefficients with the highest magnitudes are transmitted first. Since all of these coding schemes transmit bits in decreasing bit plane order, this ensures that the transmission is progressive. Such a transmission scheme makes it possible for the bit stream to be embedded, i.e., a single coded file can used to decode the image at various rates less than or equal to the coded rate, to give the best reconstruction possible with the particular coding scheme.

With these desirable features of excellent performance and low complexity, along with others such as embedded coding and progressive transmission, these scalar quantized significance testing schemes have recently become very popular in the search for practical, fast and efficient image coders, and in fact, have become the basis for serious consideration for future image compression standards.

4. Various Coding Methods of Wavelet Coefficients

One of the most successful applications of wavelet methods is transform-based image compression(also called coding).such a coder is shown in the Fig.3 operates by transforming data to remove redundancy, then quantizing the transform coefficients(a lossy step),and finally entropy coding the quantizer output. Because of the superior energy compaction properties and correspondence with human visual system, wavelet compression methods have produced superior objective and subjective results. since wavelet basis consists of functions with both short support(for high frequencies)and long support(for low frequencies),large smooth areas of an image may be represented with very few bits, and details are added where it is needed.

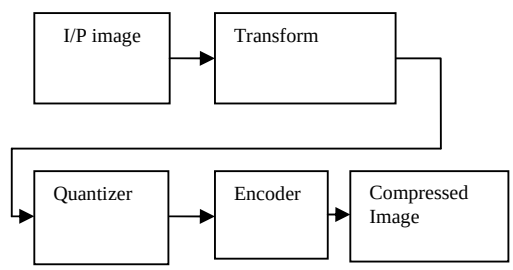


Fig. 3 Block Diagram of a Transform based coder

The decoder side is the reverse of the coder side. For Coding side we are going to discuss the following types EZW,SPIHT,SPECK,EBCOT,WDR,ASWDR,SFQ,CREW, EPWIC

4.1 Embedded Zero tree Wavelet (EZW) Coding

The EZW algorithm [13], [12] was one of the first algorithms to show the full power of wavelet-based image compression. It was introduced in the groundbreaking paper of Shapiro [13].

An EZW encoder [13] is an encoder specially designed to use with *wavelet transforms*. The EZW encoder was originally designed to operate on images (2D-signals) but it can also be used on other dimensional signals.

The EZW encoder is based on *progressive encoding* to compress an image into a bit stream with increasing

accuracy. This means that when more bits are added to the stream, the decoded image will contain more detail. Progressive encoding is also known as *embedded encoding*, which explains the E in EZW.
The EZW encoder is based on two important observations:

1. Natural images in general have a low pass spectrum. When an image is wavelet transformed the energy in the sub bands decreases as the scale decreases (low scale means high resolution), so the wavelet coefficients will, on average, be smaller in the higher sub bands than in the lower sub bands. This shows that progressive encoding is a very natural choice for compressing wavelet transformed images, since the higher sub bands only add detail;
2. Large wavelet coefficients are more important than small wavelet coefficients.

These two observations are exploited by encoding the wavelet coefficients in decreasing order, in several passes. For every pass a threshold is chosen against which all the wavelet coefficients are measured. If a wavelet coefficient is larger than the threshold it is encoded and removed from the image, if it is smaller it is left for the next pass. When all the wavelet coefficients have been visited the threshold is lowered and the image is scanned again to add more detail to the already encoded image. This process is repeated until all the wavelet coefficients have been encoded completely or another criterion has been satisfied (maximum bit rate for instance). We shall describe EZW in some detail because a solid understanding of it will make it much easier to comprehend the other algorithms we shall be discussing. These other algorithms build upon the fundamental concepts that were first introduced with EZW.
A zerotree is a quad-tree of which all nodes are equal to or smaller than the root. The tree is coded with a single symbol and reconstructed by the decoder as a quad-tree filled with zeroes. The root has to be smaller than the threshold against which the wavelet coefficients are currently being measured. The EZW encoder exploits the zerotree based on the observation that wavelet coefficients decrease with scale. The zerotree is based on the hypothesis that if a wavelet coefficient at a coarse scale is insignificant with respect to a given threshold t_0, then all wavelet coefficients of the same orientation in the same spatial location at a finer scales are likely to be insignificant with respect to t_0. The idea is to define a tree of zero symbols which starts at a root which is also zero and labeled as end-of-block. The EZW algorithm encodes the tree structure so obtained. This results in bits that are generated in order of importance, yielding a fully embedded code. The main advantage of this encoding is that the encoder can terminate the encoding at any point, thereby allowing a target bit rate to be met exactly.
To arrive at a perfect reconstruction, the process is repeated after lowering the threshold, until the threshold has become smaller than the smallest coefficient to be transmitted. Similarly, the decoder can also stop decoding at any point resulting in the image that would have been produced at the rate of the truncated bit stream. One important thing is however still missing: the transmission of the coefficient positions. Indeed, without this information the decoder will not be able to reconstruct the encoded signal (although it can perfectly reconstruct the transmitted bit stream). It is in the encoding of the positions where the efficient encoders are separated from the inefficient ones.

EZW encoding uses a predefined scan order to encode the position of the wavelet coefficients Through the use of zero-trees many positions are encoded implicitly. Several scan orders are possible, as long as the lower sub bands are completely scanned before going on to the higher sub bands.

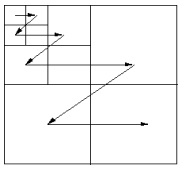


Fig.4. Scanning Order

The scan order, as shown in Fig. 4, seems to be of some influence of the final compression result. The algorithm produces excellent results without any pre-stored tables or codebooks, training, or prior knowledge of the image source. The Table.2 shows the result of compression ratio and PSNR value using EZW algorithm for Lena and Barbara image of size (512 X 512).

The unavoidable artifacts produced at low bit rates using this method are typical of wavelet coding schemes coded to the same PSNRs. However, subjectively these are not objectionable as the blocking effects typical of block transform coding schemes.
EZW encoder does not really compress anything; it only reorders wavelet coefficients in such a way that they can be compressed very efficiently.

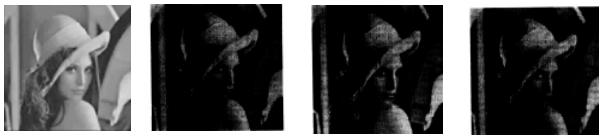
Table 2. Compression Ratio & PSNR Results

For 512 X 512 Lena

Rate	Compression Ratio	MSE	PSNR
1.0	8	7.21	39.55
0.5	16	15.32	36.28
0.25	32	31.33	33.17
0.125	64	61.67	30.23
0.0625	128	114.5	27.54
0.03125	256	188.27	25.38

For 512 X 512 Barbara:

Rate	Compression Ratio	MSE	PSNR
1.0	8	19.92	35.14
0.5	16	57.57	30.53
0.25	32	136.8	26.77
0.125	64	257.1	24.03
0.0625	128	318.5	23.10
0.03125	256	416.2	21.94



(a) (b) (c) (d)

Fig. 5. EZW coding on Lena 512X512 (a) original image (b) For 1 bpp (c) For 0.5 bpp (d) 0.25 bpp

Because of the above mentioned reason, An EZW encoder should therefore always be followed by a symbol encoder, for instance an arithmetic encoder.
The next scheme, called SPIHT, is an improved form of EZW which achieves better compression and performance than EZW.

4.2 Set Partitioning in Hierarchical Trees (SPIHT) Coding

The SPIHT coder [1], [2] is a highly refined version of the EZW algorithm and is a powerful image compression algorithm that produces an embedded bit stream from which the best reconstructed images in the *mean square error* sense can be extracted at various bit rates. Some of the best results—highest PSNR values for given compression ratios — for a wide variety of images have been obtained with SPIHT. Hence, it has become the benchmark state-of-the-art algorithm for image compression [22].

4.2.1 Set partitioning sorting algorithm

One of the main features of the SPIHT algorithm is that the ordering data is not explicitly transmitted. Instead, it is based on the fact that the execution path of any algorithm is defined by the results of the comparisons of its branching points. So, if the encoder and decoder have the same sorting algorithm, then the decoder can duplicate the encoder's execution path if it receives the results of the magnitude comparisons, and the ordering information can be recovered from the execution path.
One important fact in the design of the sorting algorithm is that there is no need to sort all coefficients. Actually, an algorithm which simply selects the coefficients such that $2^n \le |c_{i,j}| \le 2^{n+1}$,with n decremented in each pass. Given n, if $|c_{i,j}| \ge 2^n$ then the coefficient is said to be *significant*; otherwise it is called *insignificant* .The sorting algorithm divides the sets of pixels into partitioning subsets T_m and performs the magnitude test

$$\max_{(i,j)\in T_m}\{|c_{i,j}|\} \ge 2^n \qquad (3)$$

If the decoder receives a "no" as that answer, that is the subset is insignificant, then it knows that all coefficients in T_m are insignificant. If the answer is "yes", that is the subset is significant, then a certain rule shared by the decoder and encoder is used to partition T_m into new subsets and the significance test is then applied to the new subsets. This set division process continues until the magnitude test is done to all single coordinate significant subsets in order to identify each significant coefficient.
To reduce the number of magnitude comparisons, a set partitioning rule that uses an expected ordering in the hierarchy defined by the sub band pyramid, is used. The objective is to create new partitions such that subsets expected to be insignificant contain a large number of elements, and subsets expected to be significant contain only one element.
The relationship between magnitude comparisons and message bits is given by the significance function

$$S_n(T) = \begin{cases} 1, \max\limits_{(i,j)\in T_m}\{|c_{i,j}|\} \ge 2^n \\ 0, otherwise \end{cases} \qquad (3)$$

4.2.2 Spatial orientation trees

Normally, most of the image's energy is concentrated in the low frequency components. Consequently, the variance decreases as one move from the highest to the lowest of the sub band pyramid. There is a spatial self-similarity between sub bands, and the coefficients are expected to be better magnitude-ordered as one move downward in the pyramid following the same spatial orientation.
A tree structure, called spatial orientation tree, naturally defines the spatial relationship on the hierarchical pyramid.

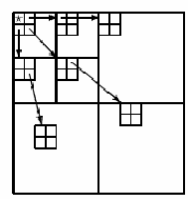


Fig.6. Parent –offspring dependencies in spatial orientation tree

Fig. 6 shows how the spatial orientation tree is defined in a pyramid constructed with recursive four-band splitting. Each node of the tree corresponds to a pixel, and is identified by the pixel coordinate. Its direct descendants (offspring) correspond to the pixels of the same spatial orientation in the next finer level of the pyramid. The tree is defined in such a way that each node has either no offspring or four off-springs, which always form a group of 2X2 adjacent pixels. The pixels in the highest level of the pyramid are the tree roots and are also grouped in 2X2 adjacent pixels. However, their offspring branching is different, and in each group one of them (indicated by the star in Fig) has no descendants. Parts of the spatial orientation trees are used as the partitioning subsets in the sorting.
With this algorithm the rate can be precisely controlled because the transmitted information is formed of single bits. The encoder can estimate the progressive distortion reduction and stop at a desired distortion value.
In the algorithm all branching conditions based on the significance data S_n, which can only be calculated with the knowledge of $c_{i,j}$ are output by the encoder. Thus, to obtain the desired decoder's algorithm, which duplicates

the encoder's execution path as it sorts the significant coefficients, the words output by input in the algorithm need to be replaced. The ordering information is recovered when the coordinates of the significant coefficients are added to the end of the LSP, that is, the coefficients pointed by the coordinates in the LSP are sorted. But whenever the decoder inputs data, its three control lists (LIS, LIP, and LSP) are identical to the ones used by the encoder at the moment it outputs that data, which means that the decoder indeed recovers the ordering from the execution path. It is easy to see that with this scheme coding and decoding have the same computational complexity.

An additional task done by decoder is to update the reconstructed image. For the value of *n* when a coordinate is moved to the LSP, it is known that $2^n \le |c_{i,j}| \le 2^{n+1}$. So, the decoder uses that information, plus the sign bit that is input just after the insertion in the LSP, to set $\hat{c}_{i,j} = \pm 1.5 * 2^n$. Similarly, during the refinement pass the decoder adds or subtracts 2^{n-1} to $\hat{c}_{i,j}$ when it inputs the bits of the binary representation of $|c_{i,j}|$. In this manner the distortion gradually decreases during both the sorting and refinement passes.

4.2.3 Features of SPIHT

The SPIHT method is not a simple extension of traditional methods for image compression, and represents an important advance in the field. The method provides the following:

- good image quality, high PSNR, especially for color images;
- it is optimized for progressive image transmission;
- produces a fully embedded coded file;
- simple quantization algorithm;
- fast coding/decoding (nearly symmetric);
- has wide applications, completely adaptive;
- Can be used for lossless compression.
- can code to exact bit rate or distortion;
- Efficient combination with error protection.

What makes SPIHT really outstanding is that it yields all those qualities simultaneously.

Table 3 gives the Compression ratio and PSNR results for SPIHT algorithm. It can be seen that the compression ratio increase, when the levels of decomposition is increased. This is because, when the levels of decomposition are increased, coefficients with higher magnitude concentrate mostly on the root levels. Also most of the coefficients will have low magnitudes. These coefficients require only less number of bits to be transmitted. Hence the compression ratio will increase when decomposition level is increased. But the resolution of the reconstructed image will reduce for higher decomposition levels.

The perceptual image quality, however, is not guaranteed to be optimal, as seen from Fig. 7, since the coder is not designed to explicitly consider the human visual system (HVS) characteristics. Extensive HVS research has shown that there are three perceptually significant activity regions in an image: *smooth*, *edge*, and *textured* or *detailed* regions [20]. By incorporating the differing sensitivity of the HVS to these regions in image compression schemes such as SPIHT, the perceptual quality of the images can be improved at all bit rates.

Table. 3. Compression ratio & PSNR Results using SPIHT for Lena 256 x 256

LENA	LEVEL	BITPLANES DISCARDED	CR	PSNR
256x256	3	3	6.57	31.28
256x256	3	5	13.03	26.81
256x256	4	3	9.44	29.00
256x256	4	5	28.38	25.87
256x256	5	3	10.38	26.76
256x256	5	5	38.94	24.66

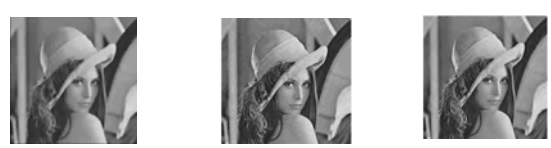
Compression ratio & PSNR Results for Barbara

BARBARA	LEVEL	BITPLANES DISCARDED	CR	PSNR
256x256	3	3	4.92	28.01
256x256	3	5	12.83	23.76
256x256	4	3	6.32	26.67
256x256	4	5	28.07	23.11
256x256	5	3	6.73	25.68
256x256	5	5	38.77	22.58

Compression ratio & PSNR Results for cameraman

CAMERA	LEVEL	BITPLANES DISCARDED	CR	PSNR
256x256	3	3	5.3053	29.66
256x256	3	5	11.5411	25.81
256x256	4	3	6.3262	26.67
256x256	4	5	22.3606	25.17
256x256	5	3	7.9141	27.36
256x256	5	5	29.3538	24.70

Efficiency of the algorithm can be improved by entropy-coding its output, but at the expense of a larger coding/decoding time. On the other hand, the significance values are not equally probable, and there is a statistical dependence between $S_n(i, j)$ and $S_n(D(i, j))$ and also between the significance of adjacent pixels.



(a) (b) (c)

Fig. 7. SPIHT results (a) Original image (b) levels of decomposition=3, 0.7 bpp (c) levels of decomposition= 9, 0.1 bpp

The need for reducing the number of lists used in this scheme led to the forming of the next algorithm, called SPECK.

4.3 Set Partitioned Embedded bloCK coder (SPECK)

The image coding scheme the SPECK [2], is different from some of the above-mentioned schemes in that it does not use trees which span, and exploit the similarity, across different sub bands; rather, it makes use of sets in the form of blocks. The main idea is to exploit the clustering of energy in frequency and space in hierarchical structures of transformed images.

The SPECK algorithm [2] can be said to belong to the class of scalar quantized significance testing schemes. It has its roots primarily in the ideas developed in the SPIHT, and few block coding image coding algorithms.

4.3.1 Features of the coder

The SPECK image coding scheme has all the properties characteristic of scalar quantized significance testing schemes.

In particular, it exhibits the following properties:

- It is completely embedded - a single coded bit stream can be used to decode the image at any rate less than or equal to the coded rate, to give the best reconstruction
- of the image possible with the particular coding scheme.
- It employs progressive transmission - source samples are coded in decreasing order of their information content.
- It has low computational complexity - the algorithm is very simple, consisting mainly of comparisons, and does not require any complex computation.
- It has low dynamic memory requirements - at any given time during the coding process, only one connected region (lying completely within a subband) is processed. Once this region is processed, the next region is then considered for processing.
- It has fast encoding/decoding - this is due to the low-complexity nature of the algorithm.
- It has efficient performance - its efficiency is comparable to the other low-complexity algorithms available today.

4.3.2 Coding method

Consider an image *X* which has been adequately transformed using an appropriate subband transformation (most commonly, the discrete wavelet transform). The transformed image is said to exhibit a hierarchical pyramidal structure defined by the levels of decomposition, with the topmost level being the root. The finest pixels lie at the bottom level of the pyramid while the coarsest pixels lie at the top (root) level.

The SPECK algorithm makes use of rectangular regions of image. These regions or sets, henceforth referred to as sets of type *S*, can be of varying dimensions. The dimension of a set *S* depends on the dimension of the original image and the subband level of the pyramidal structure at which the set lies. During the course of the algorithm, sets of various sizes will be formed, depending on the characteristics of pixels in the original set. A set of size 1 consists of just one pixel. The other type of sets used in the SPECK algorithm is referred to as sets of type *I*. These sets are obtained by chopping off a small square region from the top left portion of a larger square region. A typical set *I* is illustrated in figure below. Two linked lists: LIS - List of Insignificant Sets, and LSP - List of Significant Pixels are maintained. The former contains sets of type *S* of varying sizes which have not yet been found significant against a threshold *n* while the latter obviously contains those pixels which have tested significant against *n*. Two types of set partitioning are used in SPECK. They are quad tree partitioning and octave band partitioning.

The motivation for quad tree partitioning of sets, as shown in Fig. 9, is to zoom in quickly to areas of high energy in the set *S* and code them first.The idea behind octave band partitioning scheme, shown in Fig.10, is to exploit the hierarchical pyramidal structure of the subband de-composition, where it is more likely that energy is concentrated at the top most levels of the pyramid and as one goes down the pyramid, the energy content decreases gradually.

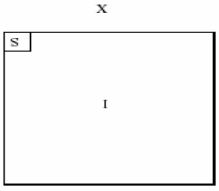


Fig.8. Partitioning of Image X into sets S & I

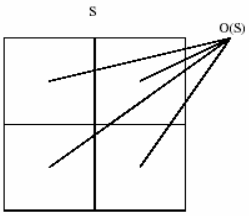


Fig.9. Quad tree Partitioning of set S

If a set *I* is significant against some threshold *n*, it is more likely that the pixels that cause *I* to be significant lie in the top left regions of *I*. These regions are decomposed into sets of type *S*, and are put next in line for processing. In this way, regions that are likely to contain significant pixels are grouped into relatively smaller sets and processed first, while regions that are likely to contain insignificant pixels are grouped into a large set.

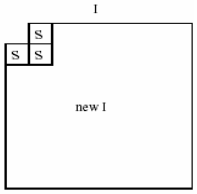


Fig.10 . Octave band Partitioning of Set I

Partitioning a set S into four off springs $O(S)$ (i.e., forming sets S of a new reduced size) is equivalent to going down the pyramid one level at the corresponding finer resolution. Hence, the size of a set S for an arbitrary image corresponds to a particular level of the pyramid.

The decoder uses the same mechanism as the encoder. It receives significance test results from the coded bit stream and builds up the same list structure during the execution of the algorithm. Hence, it is able to follow the same execution paths for the significance tests of the different sets, and reconstructs the image progressively as the algorithm proceeds. It can be seen that SPECK gives higher compression ratios. This shows advantage of processing sets in the form of blocks rather than in the form of spatial orientation trees. The reconstructed images for these compression ratios are giving appreciable resolution for the same decomposition levels and bitplanes discarded as compared with SPIHT. Fig.11. shows reconstructed images using SPECK algorithm for various values of decomposition levels and bitplanes discarded.

Table 4. Compression ration & PSNR Results
SPECK Results for Lena 256 x 256

LENA	LEVEL	BITPLANES DISCARDED	COMPRESSION RATIO	PSNR
256x256	3	3	11.23	31.59
256x256	4	3	11.76	31.56
256x256	5	3	11.86	31.53
256x256	3	5	41.25	23.93
256x256	4	5	52.05	24.08
256x256	5	5	54.60	24.16

SPECK Results for Lena 512 x 512

LENA	LEVEL	BITPLANES DISCARDED	CR	PSNR
512x512	3	3	13.35	32.34
512x512	4	3	14.39	32.25
512x512	5	3	15.35	32.12
512x512	3	5	49.82	25.31
512x512	4	5	71.94	25.66
512x512	5	5	78.50	25.51

We can see that images encoded with low decomposition levels is reconstructed with good resolution and for high decomposition levels, the resolution is reduced slightly.For high decomposition levels, most of the coefficients are of low magnitude,

SPECK Results for Barbara 512 x 512

BARBARA	LEVEL	BITPLANES DISCARDED	COMPRESSION RATIO	PSNR
256x256	3	3	7.55	26.83
256x256	4	3	7.73	26.88
256x256	5	3	8.13	26.89
256x256	3	5	46.22	19.85
256x256	4	5	58.63	19.84
256x256	5	5	61.24	19.62

SPECK Results for Camera man 256 x 256

CAMERA MAN	LEVEL	BITPLANES DISCARDED	COMPRESSION RATIO	PSNR
256x256	3	3	7.87	31.78
256x256	4	3	8.24	31.78
256x256	5	3	8.31	31.76
256x256	3	5	31.14	23.54
256x256	4	5	38.47	23.59
256x256	5	5	40.22	23.68

Hence for a certain number of bit planes discarded, the percentage error for low magnitude number is high. But for high magnitude numbers, it is negligible. Therefore with less decomposition levels the resolution of the reconstructed image is better than in SPIHT as shown in Fig. 12.

4.4 Embedded Block Coding with Optimized Truncation (EBCOT)

The EBCOT algorithm [24] uses a wavelet transform to generate the subband coefficients which are then quantized and coded. Although the usual dyadic wavelet decomposition is typical, other "packet" decompositions are also supported and occasionally preferable. The original image is represented in terms of a collection of subbands, which may be organized into increasing resolution levels. The lowest resolution level consists of the single LL subband. Each successive resolution level contains the additional subbands, which are required to reconstruct the image with twice the horizontal and vertical resolution.

Fig.11. Reconstructed Images (a) Original Image (b) Bit plane discarded=3, levels of decomposition=3 (c) Bit plane discarded=3, levels of decomposition=5 (d) Bit plane discarded=5, levels of decomposition=4(e) Bit plane discarded=5, levels of decomposition=5

Fig 12. Comparison between (a) SPIHT and (b) SPECK using Lena , for 3 levels of decomposition and 3 bitplanes discarded

The EBCOT algorithm is related in various degrees to much earlier work on scalable image compression [24]. A key advantage of scalable compression is that the target bit-rate or reconstruction resolution need not be known at the time of compression. Another advantage of practical significance is that the image need not be compressed multiple times in order to achieve a target bit-rate, as is common with the existing JPEG compression standard. EBCOT partitions each subband into relatively small blocks of samples and generates a separate highly scalable bit-stream to represent each so-called code-block. The algorithm exhibits state-of-the-art compression performance while producing a bit-stream with an unprecedented feature set, including resolution and SNR scalability together with a random access property. The algorithm has modest complexity and is extremely well suited to applications involving remote browsing of large compressed images. The first column of PSNR results, in Table 5, corresponds to the well known SPIHT [1] algorithm with arithmetic coding. The remaining columns are obtained with the EBCOT algorithm, in all cases, the popular Daubechies 9/7 bi-orthogonal wavelet filters with a five level transform is used.

Table 5. PSNR Results For Barbara (512 X 512)

BitRate	SPIHT	EBCOT 1 layer	EBCOT 5 layer	EBCOT Generic	EBCOT Spacl
0.0625	28.38	28.30	28.30	28.10	28.27
0.125	31.10	31.22	31.20	31.05	31.22
0.25	34.11	34.28	34.29	34.16	34.40
0.5	37.21	37.43	37.41	37.29	37.49
1.0	40.41	40.61	40.57	40.48	40.49

For EBCOT, code-blocks of size 64 X 64 with sub-blocks of size 16 X 16 are used. The EBCOT bit-stream is composed of a collection of quality layers and that SNR scalability is obtained by discarding unwanted layers. The second column in the table corresponds to a bit-stream with only one layer, so that the overall bit-stream is not SNR scalable. Results in this case are obtained by generating a separate compressed bit-stream for each of the relevant bit-rates. Each of the remaining columns is obtained by truncating a single bit-stream to the relevant bit-rates. The third column corresponds to a limited form of SNR scalability in which there are only five quality layers, optimized for each of the target bit-rates in the table; this may be sufficient for some applications. The fourth column corresponds to the extreme case in which 50 separate layers are included in the bit-stream spanning bit-rates ranging from approximately 0.05 bpp to 2.0 bpp.

The EBCOT images in Fig. 13. , exhibit substantially less ringing around edges and superior rendition of texture; some details preserved in the EBCOT images are completely lost by the SPIHT algorithm. In fact, for this image we find that the image quality obtained using EBCOT at 0.2 bpp is comparable to that obtained using SPIHT at 0.4 bpp. As might be expected, performance decreases as more layers are added to the bit-stream, because the overhead associated with identifying the contributions of each code-block to each layer grows. Nevertheless, performance continues to be competitive

(a) (b)

Fig .13.Comparison between (a) SPIHT (b) EBCOT at 0.15 bpp

with respect to state-of-the-art compression algorithms, significantly outperforming the common reference, SPIHT.

4.5 Wavelet Difference Reduction (WDR)

One of the defects of SPIHT is that it only implicitly locates the position of significant coefficients. This makes it difficult to perform operations which depend on the position of significant transform values, such as region selection on compressed data. Region selection, also known as region of interest (ROI), means a portion of a compressed image that requires increased resolution. This can occur, for, example, with a portion of a low resolution medical image that has been sent at a low bpp rate in order to arrive quickly.

Such compressed data operations are possible with the WDR algorithm of Tian and Wells [27]-[30]. The term difference reduction refers to the way in which WDR encodes the locations of significant wavelet transform values.

Although WDR will not produce higher PSNR values than SPIHT, as observed from Table 4, it can produce perceptually superior images, especially at high compression rates.

The only difference between WDR and bit-plane encoding is the significant pass. In WDR, the output from the significance pass consists of the signs of significant values along with sequences of bits which concisely describe the precise locations of significant values

4.6 Adaptively Scanned Wavelet Difference Reduction (ASWDR)

Though WDR produces better perceptual results than SPIHT, there is still room for improvement. One such algorithm is the ASWDR algorithm of Walker [28]. The adjective adaptively scanned refers to the fact that this algorithm modifies the scanning order used by WDR in order to achieve better performance.

ASWDR adapts the scanning order so as to predict locations of new significant values. If a prediction is correct, then the output specifying that location will just be the sign of the new significant value- the reduced binary expansion of the number of steps will be empty. Therefore, a good prediction scheme will significantly reduce the coding output of WDR. The prediction method used by ASWDR is: if w (m) is significant for threshold T, then the values of the children of m are predicted to be significant for half threshold T/2. For many natural images, this prediction method is reasonably good.

High compression ratio images are used in reconnaissance and in medical applications, where fast transmission and ROI are employed, as well as multi-resolution detection. Table. 6 shows the improved PSNR values for ASWDR compared to WDR. The WDR and ASWDR algorithms do allow for ROI while SPIHT does not. Furthermore, their superior performance in displaying edge details at low bit rates facilitates multi-resolution detection. Fig.14. shows magnifications of 128:1 and 64:1 compressions of the "Lena" image. At 0.0625 bpp, the WDR compression does a better job in preserving the shape of Lena's nose and in retaining some of the striping in the band around her hat. Similar remarks apply to the 0.125 bpp compressions. SPIHT, however, does a better job in preserving parts of Lena's eyes.

The ASWDR compressions better preserve the shape of Lena's nose and details of her hat, and show less distortion along the side of her left cheek (especially for the 0.125 bpp case).

Some other recently developed wavelet based image coding schemes are briefed below.

Table 6. PSNR Results

Image/Method	SPIHT	WDR	ASWDR
Lena,0.5 bpp	37.09	36.45	36.67
Lena, 0.25 bpp	33.85	33.39	33.64
Lena,0.125 bpp	30.85	30.42	30.61
Goldhill, 0.5 bpp	33.10	32.70	32.85
Goldhill, 0.25 bpp	30.49	30.33	30.34
Goldhill, 0.125 bpp	28.39	28.25	28.23
Barbara, 0.5 bpp	31.29	30.68	30.87
Barbara, 0.25 bpp	27.47	26.87	27.03
Barbara, 0.125 bpp	24.77	24.30	24.52
Airfield, 0.5 bpp	28.57	28.12	28.36
Airfield, 0.25 bpp	25.90	25.49	25.64
Airfield, 0.125 bpp	23.68	23.32	23.50

Fig.14. SPIHT, WDR, ASWDR Compressions (a) – (c) 0.0625 bpp , 128:1 (d) – (f) 0.125 bpp , 64:1

4.7 .Space –Frequency Quantization (SFQ)

SFQ [31] for Wavelet Image Coding belongs to a new class of image coding algorithms coupling standard scalar quantization of frequency coefficients with tree structured quantization Its good performance appears to confirm the promised efficiencies of hierarchical representation. This technique exploits both spatial and frequency compaction property of the wavelet transform through the use of two simple quantization modes. To exploit the spatial compaction property, a symbol is defined, that indicates that a spatial region of high frequency coefficients has zero value. Application of this symbol is referred to as zero-tree quantization, because it will involve setting to zero a tree-structured set of wavelet coefficients. This is done in the first phase called Tree Pruning Algorithm. In the next phase called Predicting the tree, the relation between a spatial region in image and the tree- structured set of coefficients is exploited. Zero tree quantization can be viewed as a mechanism for pointing to the location where high frequency coefficients are clustered. Thus, this quantization mode directly exploits the spatial clustering of high frequency coefficients predicted.

For coefficients that are not set to zero by zero tree quantization, a common uniform scalar quantization, independent of coefficient frequency band is applied. Uniform quantization followed by entropy coding provides nearly optimal coding efficiency.

4.8 Embedded Predictive Wavelet Image Coder (EPWIC)

EPWIC [5] is an embedded image coder based on a statistical characterization of natural images in the wavelet transform domain. The joint distribution between pairs of coefficients at adjacent spatial locations, orientations, and scales are defined. Although the raw coefficients are nearly, uncorrelated, their magnitudes are highly correlated. A linear magnitude predictor coupled with both multiplicative and additive uncertainties, provides a reasonable description of the conditional probability densities. In EPWIC, subband coefficients are encoded one bit-plane at a time using a non-adaptive arithmetic encoder. Bit-planes are ordered using an algorithm that considers the MSE reduction per encoded bit. The overall ordering of bitplanes is determined by the ratio of their encoded variance to compressed size. The coder is inherently embedded, and should prove useful in applications requiring progressive transmission.

4.9 Compression with Reversible Embedded Wavelet (CREW)

CREW [3] is a new form of still image compression developed at the RICOH California Research Center in Menlo Park, California, is a new type of image compression system and is lossy and lossless, bi-level and continuous-tone, progressive by resolution and pixel depth, and can preserve the source image at encode and quantize for the target device at decode or transmission. It uses a new form of wavelet transform technology. It is pyramidal (similar to hierarchical) and progressive by nature. CREW was the stimulus for a new JPEG 2000 standard. CREW offers a number of features that should be expected of the compression standards of the next century including:

The features make CREW an ideal choice for applications that require high quality and flexibility for multiple input and output environments, such as, medical imagery, fixed-rate and fixed-size applications (ATM, frame store, etc.) , pre-press images ,continuous-tone

facsimile documents , image archival ,World Wide Web image or graphic type , satellite images .

Many of these applications have never used compression because the quality could not be assured, the compression rate was not high enough, or the data rate was not controllable. Three new technologies combine to make CREW possible:

- the reversible wavelet transform: non-linear filters that have exact reconstruction implemented in minimal integer arithmetic
- the embedded code stream: a method of implying quantization in the code stream
- and a high-speed, high-compression binary entropy coder

The same CREW code stream can be used for both lossless and lossy applications due to embedded quantization. The wavelet transform produces pyramidally ordered data and a natural means for interpolation. The bit-significance coding allows for bit-plane progressive transmission. Furthermore, CREW compression is idempotent and CREW encoding and decoding can be simply and efficiently implemented in either hardware or software. All of these features combine to make a flexible "device-independent" compression system

4.10 Stack- Run (SR)

SR [25] image coding is a new approach in which a 4-ary arithmetic coder is used to represent significant coefficient values and the length of zero runs between coefficients. This algorithm works by raster scanning within subbands and therefore involves much lower addressing complexity than other algorithms such as zero tree coding which requires creation and maintenance of lists of dependencies across different decomposition levels. Despite its simplicity and the fact that these dependencies are not explicitly used, this algorithm is competitive with best enhancements of zero tree coding.

5. Discussions

In this section, the various features of the main coding schemes are summarized. The performance of various coding techniques and the demerits of the same are tabulated in Table 7. The latest techniques such as EBCOT, ASWDR are performing better than its predecessors such as EZW, WDR. Each technique can be well suited with different images based upon the user requirements. The Table gives complete investigation of all the coding techniques of wavelet coefficients.

Table 7 Summary of various Coding Techniques

TYPE	FEATURES	DEMERITS
EZW	• Employs progressive and embedded transmission • Uses zerotree concept • Tree coded with single symbol • Uses predefined scanning order • Good results without prestored tables, codebooks, training	• Transmission of coefficient position is missing • No real compression • Followed by arithmetic encoder
SPIHT	• Widely used- high PSNR values for given CRs for variety of images • Quad- tree or hierarchical trees are set – partitioned • Employs spatial orientation tree structure • Keeps track of states of sets of indices by means of 3 lists: LSP, LIS, LIP • Employs progressive and embedded transmission • Superior to JPEG in perceptual image quality and PSNR	• Only implicitly locates position of significant coefficients • More memory requirements due to 3 lists • Transmitted information is formed of single bits • Suits variety of natural images • Perceptual quality not optimal
SPECK	• Does not use trees • Uses blocks- rectangular regions • Exploits clustering of energy in frequency and space • Employs progressive and embedded transmission • Low computational complexity • Employ quad tree and octave band partitioning • Low memory requirements due to 2 lists • Better PSNR than SPIHT	
EBCOT	• Supports packet decompositions also • Block based scheme • Modest complexity • Bit stream composed of a collection of quality layers • SNR scalability can be obtained • Less ringing around edges • Superior rendition of textures • Preserves edges lost by SPIHT	• As layers increase, performance decreases • Suits applications involving remote browsing of large compressed images
WDR	• Uses ROI concept • Introduced by Tian and Wells • Encodes the location of significant wavelet transform values • Better perceptual image quality than SPIHT • No searching through quad trees as in SPIHT • Suits low resolution medical images at low bpp rate • Less complex than SPIHT • Higher edge correlations than SPIHT • Better preservation of edges than SPIHT	• PSNR not higher than SPIHT
ASWDR	• Modified scanning order compared to WDR • Prediction of locations of new significant values • Dynamically adapts to the locations of edge details • Encodes more significant values than WDR	

	• PSNR better than SPIHT and WDR • Perceptual image quality better than SPIHT and slightly better than WDR • Slightly higher edge correlation values than WDR • Preserves more of the fine details • Suits high CR images like in reconnaissance and medical images	

6. Conclusions

The various wavelet based image coding schemes are discussed in this paper. Each of these schemes finds use in different applications owing to their unique characteristics. Though there a number of coding schemes available, the need for improved performance and wide commercial usage, demand newer and better techniques to be developed.

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