Chapter 3

Analog Modulation

3.1 Preview

In this chapter, we study the performance of various analog modulation-demodulation schemes, both in the presence and in the absence of additive noise. Systems studied in this chapter include amplitude-modulation (AM) schemes, such as DSB-AM, SSB-AM, and conventional AM, and angle-modulation schemes, such as frequency and phase modulation. Each member of the class of analog modulation systems is characterized by five basic properties:

1. Time-domain representation of the modulated signal
2. Frequency-domain representation of the modulated signal
3. Bandwidth of the modulated signal
4. Power content of the modulated signal
5. Signal-to-noise ratio (SNR) after demodulation

These properties are obviously not independent of one another. There exists a close relationship between time- and frequency-domain representations of signals expressed through the Fourier transform relation. Also, the bandwidth of a signal is defined in terms of its frequency characteristics.

Due to the fundamental difference between amplitude- and angle-modulation schemes, these schemes are treated separately and in different sections. We begin this chapter with the study of the simplest modulation scheme, amplitude modulation.

3.2 Amplitude Modulation (AM)

Amplitude modulation (AM), which is frequently referred to as *linear modulation*, is the family of modulation schemes in which the amplitude of a sinusoidal carrier is changed as a function of the modulating signal. This class of modulation schemes consists of DSB-AM (double-sideband amplitude modulation), conventional amplitude modulation, SSB-AM (single-sideband amplitude modulation), and VSB-AM (vestigial-sideband amplitude modulation). The dependence between the modulating signal and the amplitude of the modulated carrier can be very simple, as, for example, in the DSB-AM case, or much more complex, as in SSB-AM or VSB-AM. Amplitude-modulation systems are usually characterized by a relatively low bandwidth requirement and power inefficiency in comparison to the angle-modulation schemes. The bandwidth requirement for AM systems varies between W and $2W$, where W denotes the bandwidth of the message signal. For SSB-AM the bandwidth is W, for DSB-AM and conventional AM the bandwidth is $2W$, and for VSB-AM the bandwidth is between W and $2W$. These systems are widely used in broadcasting (AM radio and TV video broadcasting), point-to-point communication (SSB), and multiplexing applications (for example, transmission of many telephone channels over microwave links).

3.2.1 DSB-AM

In DSB-AM, the amplitude of the modulated signal is proportional to the message signal. This means that the time-domain representation of the modulated signal is given by

$$u(t) = A_c m(t) \cos(2\pi f_c t) \tag{3.2.1}$$

where

$$c(t) = A_c \cos(2\pi f_c t) \tag{3.2.2}$$

is the carrier and $m(t)$ is the message signal. The frequency-domain representation of the DSB-AM signal is obtained by taking the Fourier transform of $u(t)$ and results in

$$U(f) = \frac{A_c}{2} M(f - f_c) + \frac{A_c}{2} M(f + f_c) \tag{3.2.3}$$

where $M(f)$ is the Fourier transform of $m(t)$. Obviously, this type of modulation results in a shift of $\pm f_c$ and a scaling of $A_c/2$ in the spectrum of the message signal. The transmission bandwidth denoted by B_T is twice the bandwidth of the message signal:

$$B_T = 2W \tag{3.2.4}$$

A typical message spectrum and the spectrum of the corresponding DSB-AM modulated signal are shown in Figure 3.1.

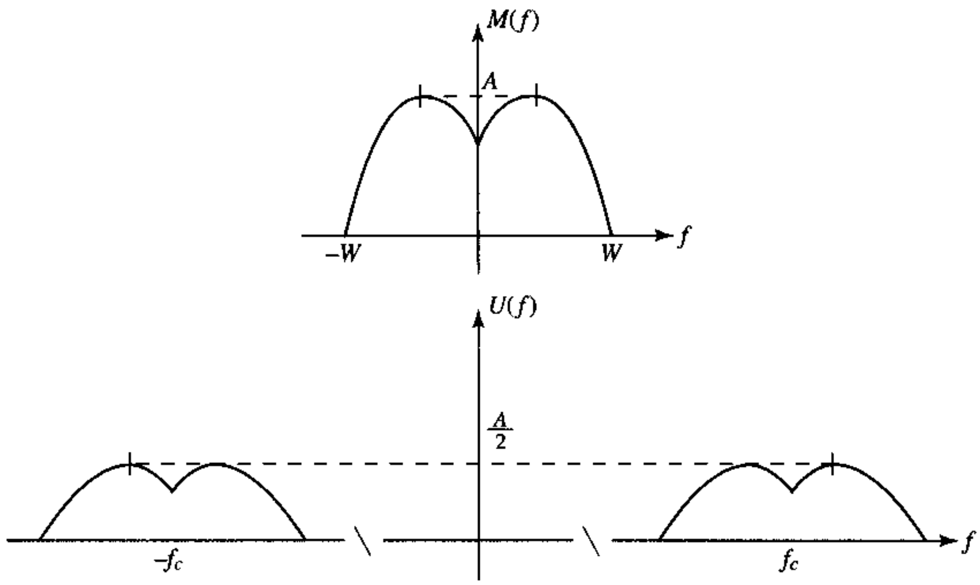


Figure 3.1 Spectra of the message and the DSB-AM modulated signal

The power content of the modulated signal is given by

$$\begin{aligned} P_u &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)\, dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m^2(t) \cos^2(2\pi f_c t)\, dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m^2(t) \frac{1+\cos(4\pi f_c t)}{2}\, dt \\ &= A_c^2 \left\{ \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{m^2(t)}{2}\, dt + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) \frac{\cos(4\pi f_c t)}{2}\, dt \right\} \\ &= A_c^2 \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{m^2(t)}{2}\, dt \qquad (3.2.5) \\ &= \frac{A_c^2}{2} P_m \qquad (3.2.6) \end{aligned}$$

where P_m is the power content of the message signal. Equation (3.2.5) follows from the fact that $m(t)$ is a lowpass signal with frequency content much less than $2f_c$, the frequency content of $\cos(4\pi f_c t)$. Therefore, the integral

$$\int_{-T/2}^{T/2} m^2(t) \frac{\cos(4\pi f_c t)}{2}\, dt \qquad (3.2.7)$$

goes to zero as $T \to \infty$. Finally, the SNR for a DSB-AM system is equal to the baseband SNR; that is, it is given by

$$\left(\frac{S}{N}\right)_o = \frac{P_R}{N_0 W} \qquad (3.2.8)$$

where P_R is the received power (the power in the modulated signal at the receiver), $N_0/2$ is the noise power spectral density (assuming white noise), and W is the message bandwidth.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.1 [DSB-AM modulation] The message signal $m(t)$ is defined as

$$m(t) = \begin{cases} 1, & 0 \le t \le \frac{t_0}{3} \\ -2, & \frac{t_0}{3} < t \le \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

This message DSB-AM modulates the carrier $c(t) = \cos 2\pi f_c t$, and the resulting modulated signal is denoted by $u(t)$. It is assumed that $t_0 = 0.15$ s and $f_c = 250$ Hz.

1. Obtain the expression for $u(t)$.
2. Derive the spectra of $m(t)$ and $u(t)$.
3. Assuming that the message signal is periodic with period $T_0 = t_0$, determine the power in the modulated signal.
4. If a noise is added to the modulated signal in part 3 such that the resulting SNR is 10 dB, find the noise power.

SOLUTION

1. $m(t)$ can be written as

$$m(t) = \Pi\left(\frac{t - t_0/6}{t_0/3}\right) - 2\Pi\left(\frac{t - t_0/2}{t_0/3}\right)$$

Therefore,

$$u(t) = \left[\Pi\left(\frac{t - 0.025}{0.05}\right) - 2\Pi\left(\frac{t - 0.075}{0.05}\right)\right]\cos(500\pi t) \tag{3.2.9}$$

2. Using the standard Fourier transform relation $\mathcal{F}[\Pi(t)] = \text{sinc}(t)$ together with the shifting and the scaling theorems of the Fourier transform, we obtain

$$\begin{aligned}\mathcal{F}[m(t)] &= \frac{t_0}{3}e^{-j\pi f t_0/3}\text{sinc}\left(\frac{t_0 f}{3}\right) - 2\frac{t_0}{3}e^{-j\pi f t_0}\text{sinc}\left(\frac{t_0 f}{3}\right) \\ &= \frac{t_0}{3}e^{-j\pi f t_0/3}\text{sinc}\left(\frac{t_0 f}{3}\right)\left(1 - 2e^{-j2\pi t_0 f/3}\right) \end{aligned} \tag{3.2.10}$$

Substituting $t_0 = 0.15$ s gives

$$\mathcal{F}[m(t)] = 0.05e^{-0.05j\pi f}\text{sinc}(0.05f)\left(1 - 2e^{-0.1j\pi f}\right) \tag{3.2.11}$$

For the modulated signal $u(t)$, we have

$$U(f) = 0.025e^{-0.05j\pi(f-f_c)}\text{sinc}\,(0.05(f-f_c))\left(1-2e^{-0.1j\pi(f-f_c)}\right)$$
$$+\,0.025e^{-0.05j\pi(f+f_c)}\text{sinc}\,(0.05(f+f_c))\left(1-2e^{-0.1j\pi(f+f_c)}\right)$$

Plots of the magnitude spectra of the message and the modulated signals are shown in Figure 3.2.

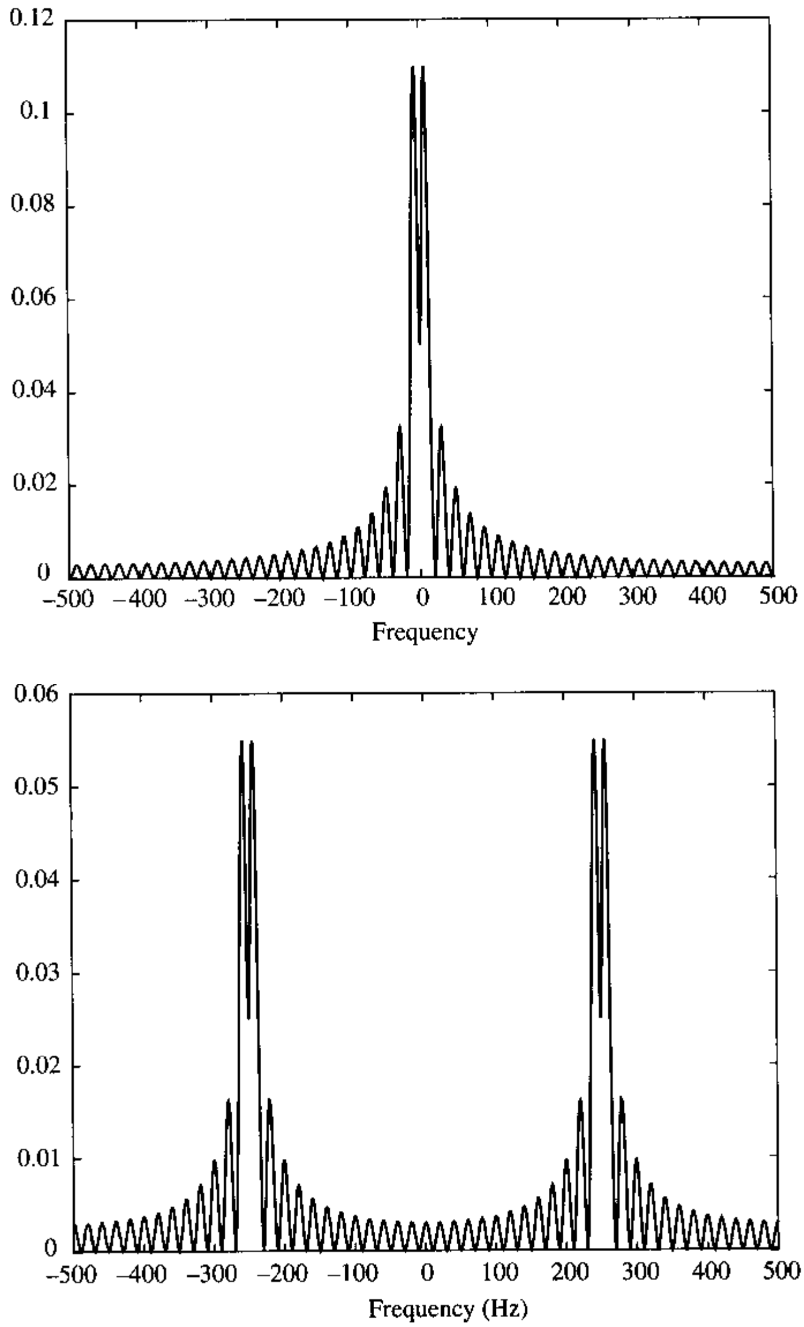


Figure 3.2 Magnitude spectra of the message and the modulated signals in Illustrative Problem 3.1

3. The power in the modulated signal is given by

$$P_u = \frac{A_c^2}{2} P_m = \frac{1}{2} P_m$$

where P_m is the power in the message signal;

$$P_m = \frac{1}{t_0} \int_0^{2t_0/3} m^2(t)\, dt = \frac{1}{t_0}\left(\frac{t_0}{3} + \frac{4t_0}{3}\right) = \frac{5}{3} = 1.666$$

and

$$P_u = \frac{1.666}{2} = 0.833$$

4. Here

$$10 \log_{10}\left(\frac{P_R}{P_n}\right) = 10$$

or $P_R = P_u = 10P_n$, which results in $P_n = P_u/10 = 0.0833$.

The MATLAB script for the preceding problem follows.

M-FILE

```
% Matlab script for Illustrative Problem 3.1.
% Matlab demonstration script for DSB-AM modulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                  % signal duration
ts=0.001;                                % sampling interval
fc=250;                                  % carrier frequency
snr=20;                                  % SNR in dB (logarithmic)
fs=1/ts;                                 % sampling frequency
df=0.3;                                  % desired freq. resolution
t=[0:ts:t0];                             % time vector
snr_lin=10^(snr/10);                     % linear SNR
% message signal
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                       % carrier signal
u=m.*c;                                  % modulated signal
[M,m,df1]=fftseq(m,ts,df);               % Fourier transform
M=M/fs;                                  % scaling
[U,u,df1]=fftseq(u,ts,df);               % Fourier transform
U=U/fs;                                  % scaling
[C,c,df1]=fftseq(c,ts,df);               % Fourier transform
f=[0:df1:df1*(length(m)-1)]-fs/2;        % freq. vector
signal_power=spower(u(1:length(t)));     % power in modulated signal
noise_power=signal_power/snr_lin;        % compute noise power
noise_std=sqrt(noise_power);             % compute noise standard deviation
```

```
noise=noise_std*randn(1,length(u));          % generate noise
r=u+noise;                                   % add noise to the modulated signal
[R,r,df1]=fftseq(r,ts,df);                   % spectrum of the signal+noise
R=R/fs;                                      % scaling
pause  % Press a key to show the modulated signal power
signal_power
pause  % Press any key to see plot of the message
clf
subplot(2,2,1)
plot(t,m(1:length(t)))
xlabel('Time')
title('The message signal')
pause  % Press any key to see a plot of the carrier
subplot(2,2,2)
plot(t,c(1:length(t)))
xlabel('Time')
title('The carrier')
pause  % Press any key to see a plot of the modulated signal
subplot(2,2,3)
plot(t,u(1:length(t)))
xlabel('Time')
title('The modulated signal')
pause   % Press any key to see plots of the magnitude of the message and the
        % modulated signal in the frequency domain.
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Spectrum of the message signal')
subplot(2,1,2)
plot(f,abs(fftshift(U)))
title('Spectrum of the modulated signal')
xlabel('Frequency')
pause  % Press a key to see a noise sample
subplot(2,1,1)
plot(t,noise(1:length(t)))
title('Noise sample')
xlabel('Time')
pause  % Press a key to see the modulated signal and noise
subplot(2,1,2)
plot(t,r(1:length(t)))
title('Signal and noise')
xlabel('Time')
pause  % Press a key to see the modulated signal and noise in freq. domain
subplot(2,1,1)
plot(f,abs(fftshift(U)))
title('Signal spectrum')
xlabel('Frequency')
subplot(2,1,2)
plot(f,abs(fftshift(R)))
title('Signal and noise spectrum')
xlabel('Frequency')
```

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.2 [DSB modulation for an almost bandlimited signal] The message signal $m(t)$ is given by

$$m(t) = \begin{cases} \text{sinc}(100t), & |t| \leq t_0 \\ 0, & \text{otherwise} \end{cases}$$

where $t_0 = 0.1$. This message modulates the carrier $c(t) = \cos(2\pi f_c t)$, where $f_c = 250$ Hz.

1. Determine the modulated signal $u(t)$.
2. Determine the spectra of $m(t)$ and $u(t)$.
3. If the message signal is periodic with period $T_0 = 0.2$ s, determine the power in the modulated signal.
4. If a Gaussian noise is added to the modulated signal such that the resulting SNR is 10 dB, find the noise power.

SOLUTION

1. We have

$$\begin{aligned} u(t) &= m(t)c(t) \\ &= \begin{cases} \text{sinc}(100t)\cos(500t), & |t| \leq 0.1 \\ 0, & \text{otherwise} \end{cases} \qquad (3.2.12) \\ &= \text{sinc}(100t)\Pi(5t)\cos(500t) \qquad (3.2.13) \end{aligned}$$

2. A plot of the spectra of $m(t)$ and $u(t)$ is shown in Figure 3.3. As the figure shows, the message signal is an almost bandlimited signal with a bandwidth of 50 Hz.

3. The power in the modulated signal is half of the power in the message signal. The power in the message signal is given by

$$P_m = \frac{1}{0.2}\int_{-0.1}^{0.1} \text{sinc}^2(100t)\,dt$$

The integral can be computed using MATLAB's quad8.m m-file, which results in $P_m = 0.0495$ and hence, $P_u = 0.0247$.

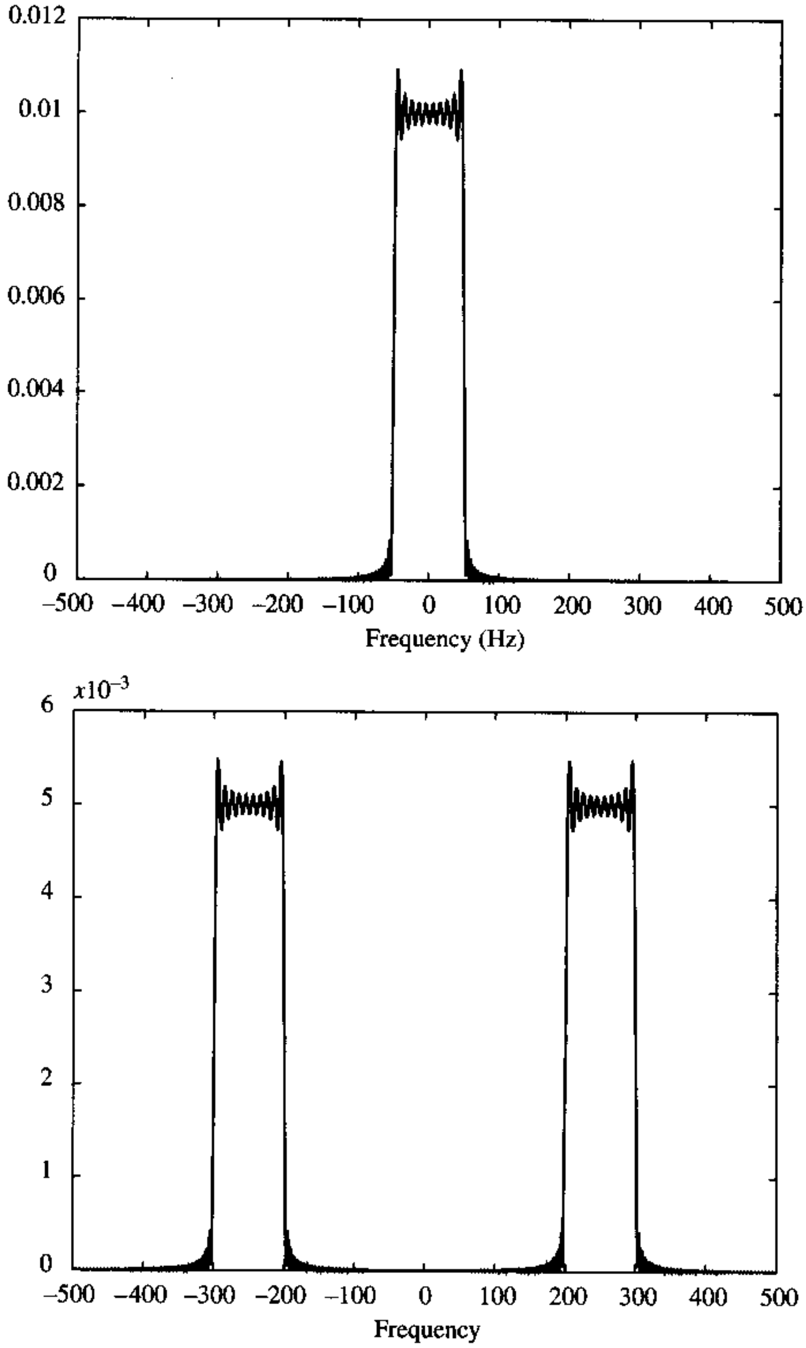


Figure 3.3 Spectra of the message and the modulated signals in Illustrative Problem 3.2

4. Here

$$10\log_{10}\left(\frac{P_R}{P_n}\right) = 10 \Longrightarrow P_n = 0.1P_R = 0.1P_U = 0.00247$$

The MATLAB script for the problem follows.

M-FILE

```
% Matlab script for Illustrative Problem 3.2.
% Matlab demonstration script for DSB-AM modulation. The message signal
% is m(t)=sinc(100t).
echo on
t0=.2;                                    % signal duration
ts=0.001;                                 % sampling interval
fc=250;                                   % carrier frequency
snr=20;                                   % SNR in dB (logarithmic)
fs=1/ts;                                  % sampling frequency
df=0.3;                                   % required freq. resolution
t=[-t0/2:ts:t0/2];                        % time vector
snr_lin=10^(snr/10);                      % linear SNR
m=sinc(100*t);                            % the message signal
c=cos(2*pi*fc.*t);                        % the carrier signal
u=m.*c;                                   % the DSB-AM modulated signal
[M,m,df1]=fftseq(m,ts,df);                % Fourier transform
M=M/fs;                                   % scaling
[U,u,df1]=fftseq(u,ts,df);                % Fourier transform
U=U/fs;                                   % scaling
f=[0:df1:df1*(length(m)-1)]-fs/2;         % frequency vector
signal_power=spower(u(1:length(t)));      % compute modulated signal power
noise_power=signal_power/snr_lin;         % compute noise power
noise_std=sqrt(noise_power);              % compute noise standard deviation
noise=noise_std*randn(1,length(u));       % generate noise sequence
r=u+noise;                                % add noise to the modulated signal
[R,r,df1]=fftseq(r,ts,df);                % Fourier transform
R=R/fs;                                   % scaling
pause % Press a key to show the modulated signal power
signal_power
pause %Press any key to see a plot of the message
clf
subplot(2,2,1)
plot(t,m(1:length(t)))
xlabel('Time')
title('The message signal')
pause % Press any key to see a plot of the carrier
subplot(2,2,2)
plot(t,c(1:length(t)))
xlabel('Time')
title('The carrier')
pause % Press any key to see a plot of the modulated signal
subplot(2,2,3)
plot(t,u(1:length(t)))
xlabel('Time')
title('The modulated signal')
pause % Press any key to see a plot of the magnitude of the message and the
      % modulated signal in the frequency domain.
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Spectrum of the message signal')
```

```
subplot(2,1,2)
plot(f,abs(fftshift(U)))
title('Spectrum of the modulated signal')
xlabel('Frequency')
pause % Press a key to see a noise sample
subplot(2,1,1)
plot(t,noise(1:length(t)))
title('Noise sample')
xlabel('Time')
pause % Press a key to see the modulated signal and noise
subplot(2,1,2)
plot(t,r(1:length(t)))
title('Signal and noise')
xlabel('Time')
pause % Press a key to see the modulated signal and noise in freq. domain
subplot(2,1,1)
plot(f,abs(fftshift(U)))
title('Signal spectrum')
xlabel('Frequency')
subplot(2,1,2)
plot(f,abs(fftshift(R)))
title('Signal and noise spectrum')
xlabel('Frequency')
```

WHAT IF?

What happens if the duration of the message signal t_0 changes; in particular, what is the effect of having large t_0's and small t_0's? What is the effect on the bandwidth and signal power?

The m-file dsb_mod.m given next is a general DSB modulator of the message signal given in vector m on a carrier of frequency f_c.

M-FILE

```
function u=dsb_mod(m,ts,fc)
%           u=dsb_mod(m,ts,fc)
%DSB_MOD    takes signal m sampled at ts and carrier
%           freq. fc as input and returns the DSB modulated
%           signal. ts << 1/2fc. The modulated signal
%           is normalized to have half the message power.
%           The message signal starts at 0.

t=[0:length(m)-1]*ts;
u=m.*cos(2*pi*t);
```

3.2.2 Conventional AM

In many respects, conventional AM is quite similar to DSB-AM; the only difference is that in conventional AM, $m(t)$ is substituted with $[1 + am_n(t)]$, where $m_n(t)$ is the normalized message signal (i.e., $|m_n(t)| \leq 1$) and a is the *index of modulation*, which is a positive constant between 0 and 1. Thus, we have

$$u(t) = A_c\,[1 + am_n(t)]\cos(2\pi f_c t) \tag{3.2.14}$$

and

$$U(f) = \frac{A_c}{2}[\delta(f - f_c) + aM_n(f - f_c) + \delta(f + f_c) + aM_n(f + f_c)] \tag{3.2.15}$$

The net effect of scaling the message signal and adding a constant to it is that the term $[1 + am_n(t)]$ is always positive. This makes the demodulation of these signals much easier by employing envelope detectors. Note the existence of the sinusoidal component at the frequency f_c in $U(f)$. This means that a (usually substantial) fraction of the transmitted power is in the signal carrier that does not really serve the transmission of information. This fact shows that compared to DSB-AM, conventional AM is a less economical modulation scheme in terms of power utilization. The bandwidth, of course, is equal to the bandwidth of DSB-AM and is given by

$$B_T = 2W \tag{3.2.16}$$

Typical frequency-domain plots of the message and the corresponding conventional AM signal are shown in Figure 3.4.

The power content of the modulated signal, assuming that the message signal is a zero-mean signal, is given by

$$P_u = \frac{A_c^2}{2}\left[1 + a^2 P_{m_n}\right] \tag{3.2.17}$$

which comprises two parts: $A_c^2/2$, which denotes the power in the carrier, and $(A_c^2/2)a^2 P_{m_n}$, which is the power in the message-bearing part of the modulated signal. This is the power that is really used to transmit the message. The ratio of the power that is used to transmit the message to the total power in the modulated signal is called the *modulation efficiency* and is defined by

$$\eta = \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}} \tag{3.2.18}$$

Since $|m_n(t)| \leq 1$ and $a \leq 1$, we always have $\eta \leq 0.5$. In practice, however, the value of η is around 0.1. The signal-to-noise ratio is given by

$$\left(\frac{S}{N}\right)_o = \eta \frac{P_R}{N_0 W} \tag{3.2.19}$$

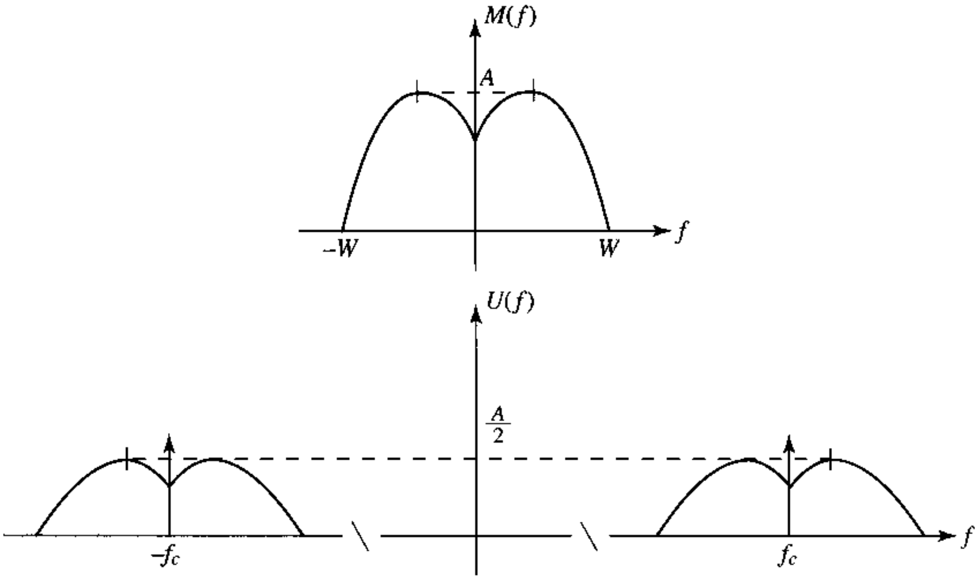


Figure 3.4 Spectra of the message and the conventional AM signal

where η is the modulation efficiency. We see that the SNR is reduced by a factor equal to η compared to a DSB-AM system. This reduction of performance is a direct consequence of the fact that a significant part of the total power is in the carrier (the deltas in the spectrum), which does not carry any information and is filtered out at the receiver.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.3 [Conventional AM] The message signal

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

modulates the carrier $c(t) = \cos(2\pi f_c t)$ using a conventional AM scheme. It is assumed that $f_c = 250$ Hz and $t_0 = 0.15$; the modulation index is $a = 0.85$.

1. Derive an expression for the modulated signal.
2. Determine the spectra of the message and the modulated signals.
3. If the message signal is periodic with a period equal to t_0, determine the power in the modulated signal and the modulation efficiency.
4. If a noise signal is added to the message signal such that the SNR at the output of the demodulator is 10 dB, find the power content of the noise signal.

SOLUTION

1. First note that $\max |m(t)| = 2$; therefore, $m_n(t) = m(t)/2$. From this, we have

$$u(t) = \left[1 + 0.85\frac{m(t)}{2}\right]\cos(2\pi f_c t)$$
$$= \left[1 + 0.425\Pi\left(\frac{t-0.025}{0.05}\right) - 0.85\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos(500\pi t)$$

A plot of the message and the modulated signal is shown in Figure 3.5.

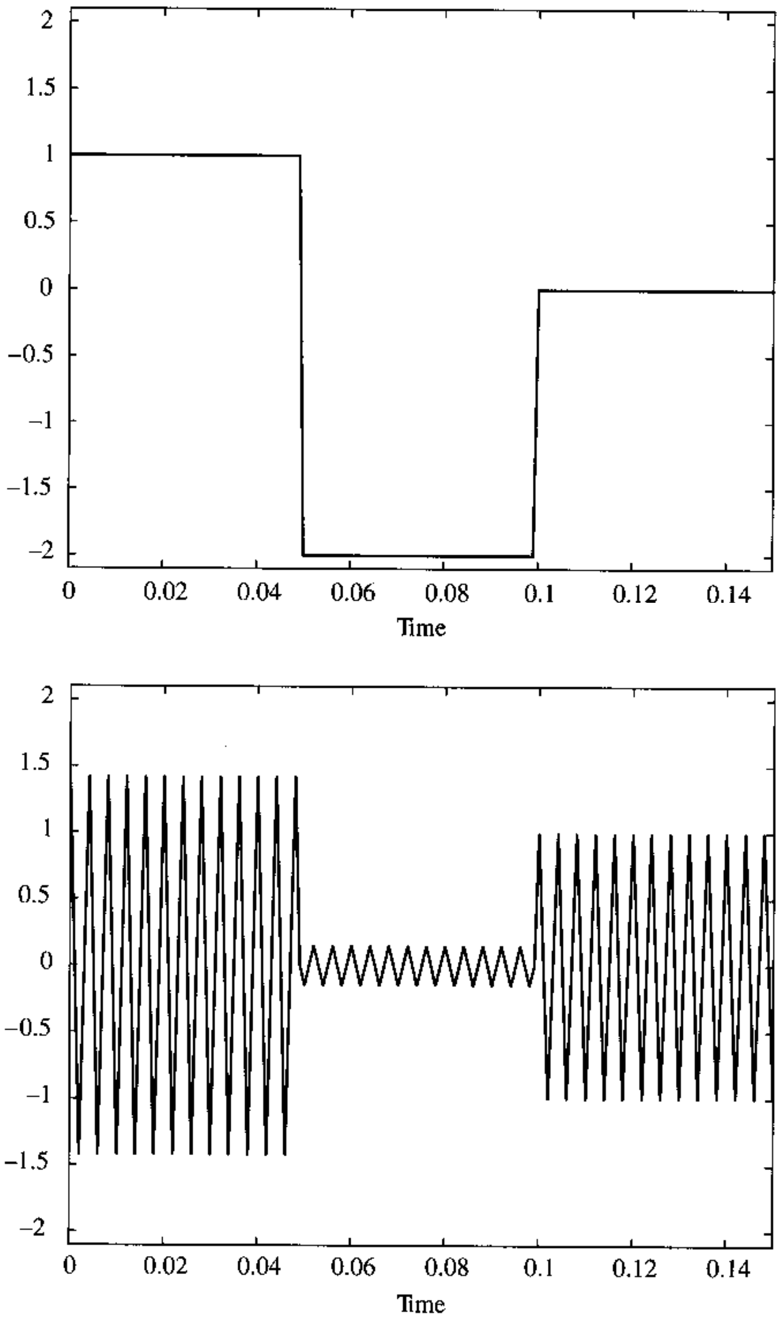


Figure 3.5 The message and the modulated signals in Illustrative Problem 3.3

2. For the message signal, we have

$$\mathcal{F}[m(t)] = 0.05e^{-0.05j\pi f}\,\text{sinc}(0.05f)\left(1 - 2e^{-0.1j\pi f}\right) \tag{3.2.20}$$

and for the modulated signal,

$$U(f) = 0.010625e^{-0.05j\pi(f-250)}\text{sinc}(0.05(f-250))\left(1 - 2e^{-0.1j\pi(f-250)}\right)$$
$$+\,0.010625e^{-0.05j\pi(f+250)}\text{sinc}(0.05(f+250))\left(1 - 2e^{-0.1j\pi(f+250)}\right)$$

Plots of the spectra of the message and the modulated signal are shown in Figure 3.6.

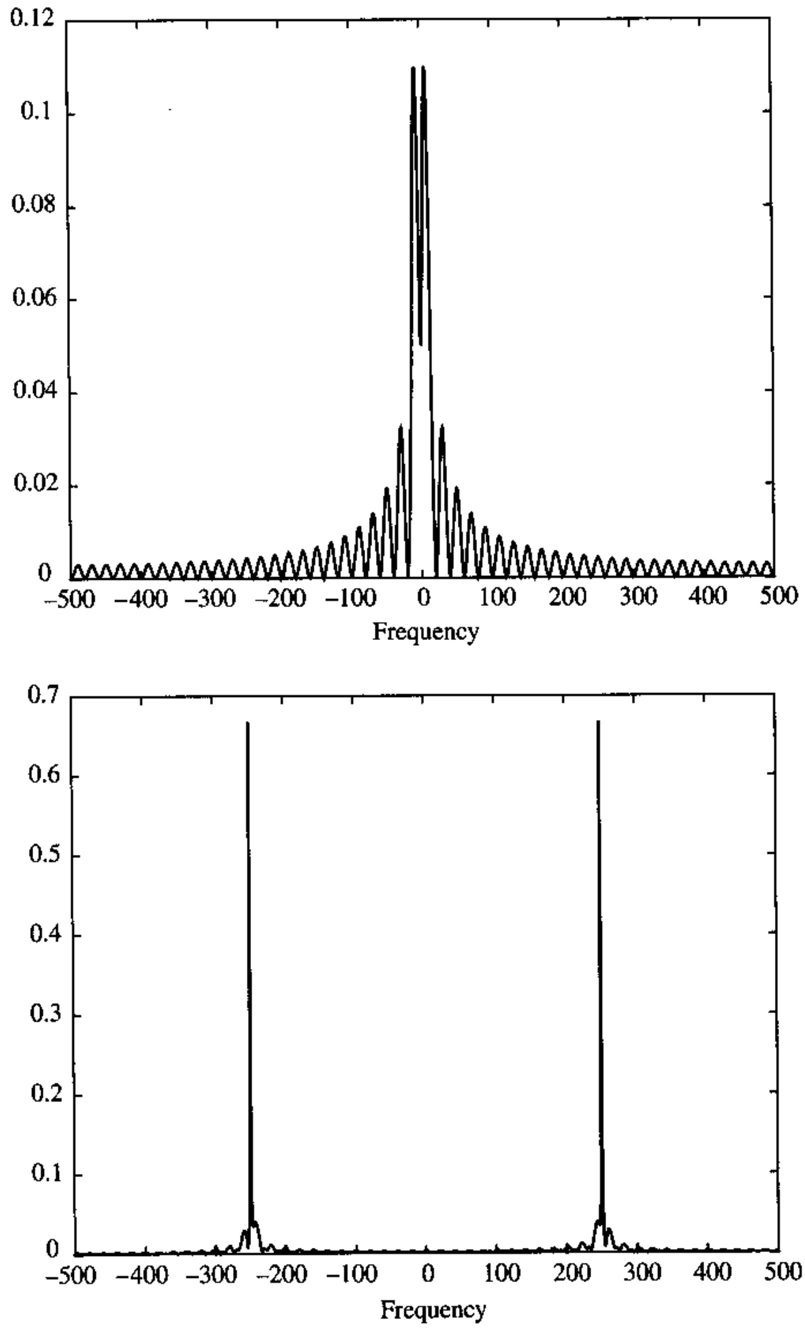


Figure 3.6 Spectra of the message and the modulated signal in Illustrative Problem 3.3

Note that the scales on the two spectra plots are different. The presence of the two delta functions in the spectrum of the modulated signal is apparent.

3. The power in the message signal can be obtained as

$$P_m = \frac{1}{0.15}\left[\int_0^{0.05} dt + 4\int_{0.05}^{0.1} dt\right] = 1.667$$

The power in the normalized message signal P_{m_n} is given by

$$P_{m_n} = \frac{1}{4}P_m = \frac{1.66}{4} = 0.4167$$

and the modulation efficiency is

$$\eta = \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}} = \frac{0.85^2 \times 0.4167}{1 + 0.85^2 \times 0.4167} = 0.2314$$

The power in the modulated signal is given by (E denotes the expected value)

$$\begin{aligned} P_u &= \frac{A_c^2}{2} E\left[1 + a m_n(t)\right]^2 \\ &= \frac{1}{2}\left(1 + 0.3010 - 1.7 \times \frac{0.025}{0.15}\right) \\ &= 0.5088 \end{aligned}$$

4. In this case,

$$10\log_{10}\left[\eta\left(\frac{P_R}{N_0 W}\right)\right] = 10$$

or

$$\eta\left(\frac{P_R}{P_n}\right) = 10$$

Substituting $\eta = 0.2314$ and $P_R = P_u = 0.5088$ yields

$$P_n = \frac{\eta P_u}{10} = 0.0118$$

COMMENT

In finding the power in the modulated signal in this problem, we could not use the relation

$$P_u = \frac{A_c^2}{2}\left[1 + a^2 P_{m_n}\right]$$

because in this problem $m(t)$ is not a zero-mean signal.

The MATLAB script for this problem is given next.

M-FILE

```
% am.m
% Matlab demonstration script for DSB-AM modulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                    % signal duration
ts=0.001;                                  % sampling interval
fc=250;                                    % carrier frequency
snr=10;                                    % SNR in dB (logarithmic)
a=0.85;                                    % modulation index
fs=1/ts;                                   % sampling frequency
t=[0:ts:t0];                               % time vector
df=0.2;                                    % required frequency resolution
snr_lin=10^(snr/10);                       % SNR
% message signal
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                         % carrier signal
m_n=m/max(abs(m));                         % normalized message signal
[M,m,df1]=fftseq(m,ts,df);                 % Fourier transform
M=M/fs;                                    % scaling
f=[0:df1:df1*(length(m)-1)]-fs/2;          % frequency vector
u=(1+a*m_n).*c;                            % modulated signal
[U,u,df1]=fftseq(u,ts,df);                 % Fourier transform
U=U/fs;                                    % scaling
signal_power=spower(u(1:length(t)));       % power in modulated signal
% power in normalized message
pmn=spower(m(1:length(t)))/(max(abs(m)))^2;
eta=(a^2*pmn)/(1+a^2*pmn);                 % modulation efficiency
noise_power=eta*signal_power/snr_lin;      % noise power
noise_std=sqrt(noise_power);               % noise standard deviation
noise=noise_std*randn(1,length(u));        % generate noise
r=u+noise;                                 % add noise to the modulated signal
[R,r,df1]=fftseq(r,ts,df);                 % Fourier transform
R=R/fs;                                    % scaling
pause  % Press a key to show the modulated signal power
signal_power
pause  % Press a key to show the modulation efficiency
eta
pause  % Press any key to see plot of the message
subplot(2,2,1)
plot(t,m(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The message signal')
pause
pause  % Press any key to see a plot of the carrier
subplot(2,2,2)
plot(t,c(1:length(t)))
```

```
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The carrier')
pause % Press any key to see a plot of the modulated signal
subplot(2,2,3)
plot(t,u(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The modulated signal')
pause % Press any key to see a plot of the magnitude of the message and the
      % modulated signal in the frequency domain.
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Spectrum of the message signal')
subplot(2,1,2)
plot(f,abs(fftshift(U)))
title('Spectrum of the modulated signal')
xlabel('Frequency')
pause % Press a key to see a noise sample
subplot(2,1,1)
plot(t,noise(1:length(t)))
title('Noise sample')
xlabel('Time')
pause % Press a key to see the modulated signal and noise
subplot(2,1,2)
plot(t,r(1:length(t)))
title('Signal and noise')
xlabel('Time')
pause % Press a key to see the modulated signal and noise in freq. domain
subplot(2,1,1)
plot(f,abs(fftshift(U)))
title('Signal spectrum')
xlabel('Frequency')
subplot(2,1,2)
plot(f,abs(fftshift(R)))
title('Signal and noise spectrum')
xlabel('Frequency')
```

The MATLAB m-file am_mode.m given next is a general conventional AM modulator.

M-FILE

```
function u=am_mod(a,m,ts,fc)
%           u=am_mod(a,m,ts,fc)
%AM_MOD     takes signal m sampled at ts and carrier
%           freq. fc as input and returns the AM modulated
%           signal. "a" is the modulation index.
%           and ts << 1/2fc.

t=[0:length(m)-1]*ts;
```

```
c=cos(2*pi*fc.*t);
m_n=m/max(abs(m));
u=(1+a*m_n).*c;
```

3.2.3 SSB-AM

SSB-AM is derived from DSB-AM by eliminating one of the sidebands. Therefore, it occupies half the bandwidth of DSB-AM. Depending on the sideband that remains, either the upper or the lower sideband, there exist two types of SSB-AM: USSB-AM and LSSB-AM. The time representation for these signals is given by

$$u(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t) \tag{3.2.21}$$

where the minus sign corresponds to USSB-AM and the plus sign corresponds to LSSB-AM. The signal denoted by $\hat{m}(t)$ is the Hilbert transform of $m(t)$, defined by $\hat{m}(t) = m(t) \star 1/\pi t$ or, in the frequency domain, by $\hat{M}(f) = -j\ \mathrm{sgn}(f)M(f)$. In other words, the Hilbert transform of a signal represents a $\pi/2$ phase shift in all signal components. In the frequency domain, we have

$$U_{\mathrm{USSB}}(f) = \begin{cases} [M(f - f_c) + M(f + f_c)], & f_c \le |f| \\ 0, & \text{otherwise} \end{cases} \tag{3.2.22}$$

and

$$U_{\mathrm{LSSB}}(f) = \begin{cases} [M(f - f_c) + M(f + f_c)], & |f| \le f_c \\ 0, & \text{otherwise} \end{cases} \tag{3.2.23}$$

Typical plots of the spectra of a message signal and the corresponding USSB-AM modulated signal are shown in Figure 3.7.

The bandwidth of the SSB signal is half the bandwidth of DSB and conventional AM and so is equal to the bandwidth of the message signal; that is,

$$B_T = W \tag{3.2.24}$$

The power in the SSB signal is given by

$$P_u = \frac{A_c^2}{4} P_m \tag{3.2.25}$$

Note that the power is half of the power in the corresponding DSB-AM signal because one of the sidebands has been removed. On the other hand, since the modulated signal has half the bandwidth of the corresponding DSB-AM signal, the noise power at the front end of the receiver is also half of a comparable DSB-AM signal, and therefore the SNR in both systems is the same; that is,

$$\left(\frac{S}{N}\right)_{\mathrm{o}} = \frac{P_R}{N_0 W} \tag{3.2.26}$$

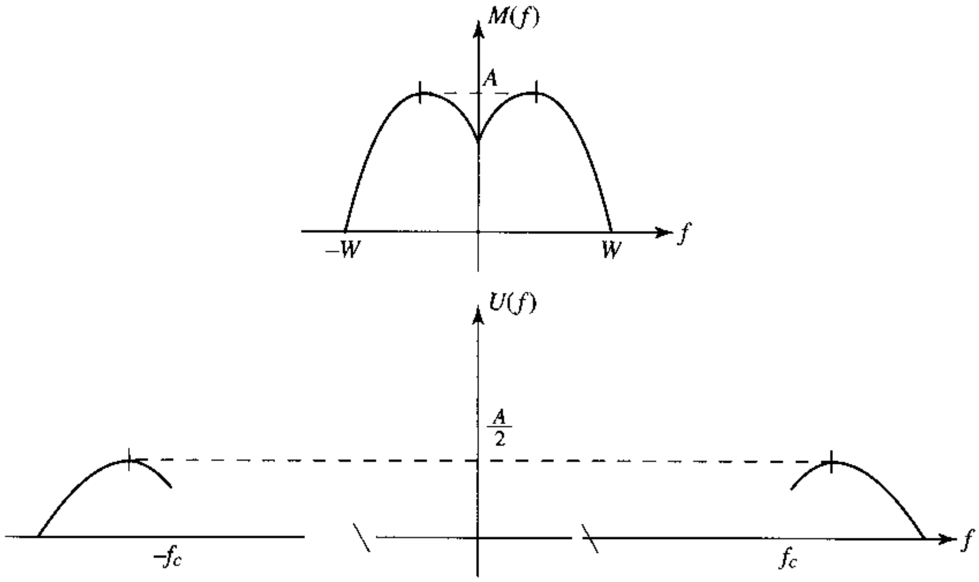


Figure 3.7 Spectra of the message and the USSB-AM signal

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.4 [Single-sideband example] The message signal

$$m(t) = \begin{cases} 1, & 0 \leq t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \leq t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

modulates the carrier $c(t) = \cos(2\pi f_c t)$ using an LSSB-AM scheme. It is assumed that $t_0 = 0.15$ s and $f_c = 250$ Hz.

1. Plot the Hilbert transform of the message signal and the modulated signal $u(t)$. Also plot the spectrum of the modulated signal.
2. Assuming the message signal is periodic with period t_0, determine the power in the modulated signal.
3. If a noise is added to the modulated signal such that the SNR after demodulation is 10 dB, determine the power in the noise.

SOLUTION

1. The Hilbert transform of the message signal can be computed using the Hilbert transform m-file of MATLAB—that is, hilbert.m. It should be noted, however, that this function returns a complex sequence whose real part is the original signal and whose

imaginary part is the desired Hilbert transform. Therefore, the Hilbert transform of the sequence m is obtained by using the command imag(hilbert(m)). Now, using the relation

$$u(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t) \tag{3.2.27}$$

we can find the modulated signal. Plots of $\hat{m}(t)$ and the spectrum of the LSSB-AM modulated signal $u(t)$ are shown in Figure 3.8.

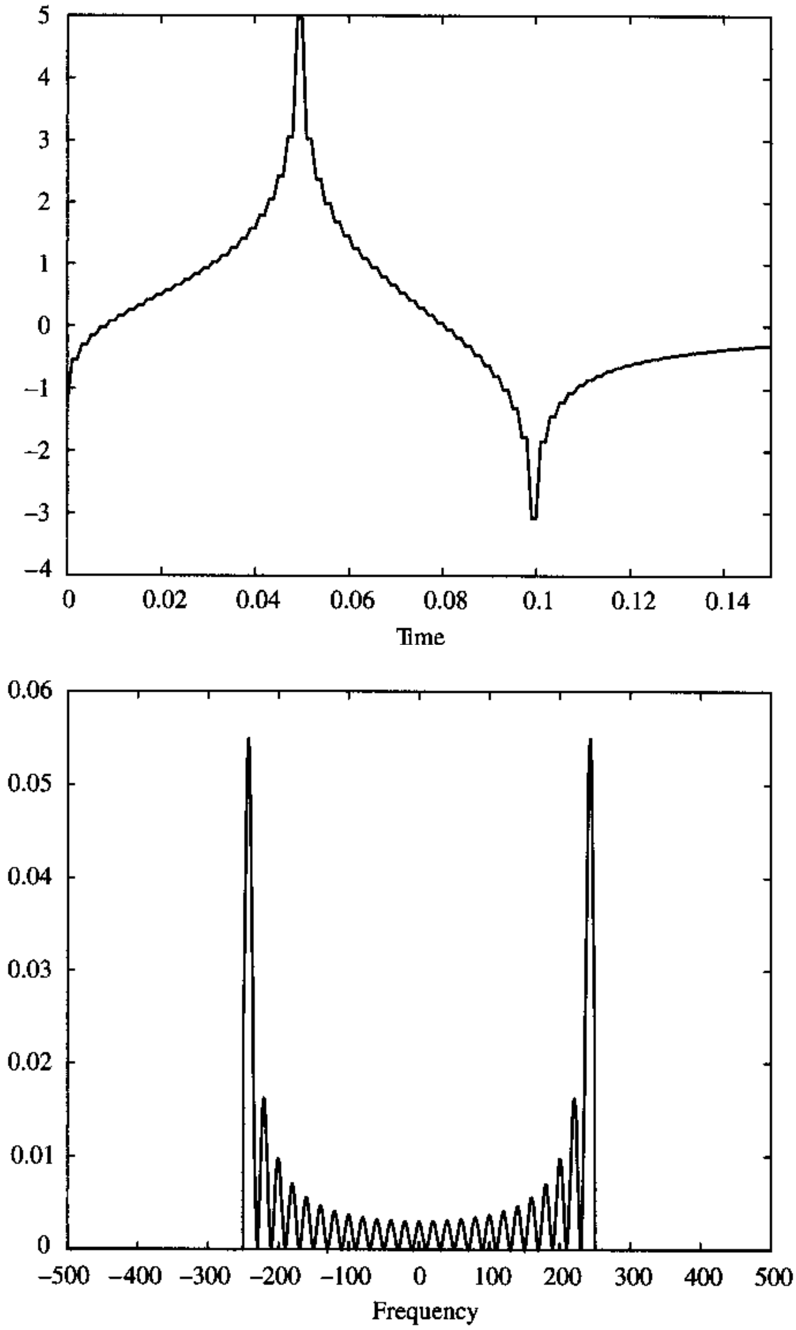


Figure 3.8 Hilbert transform and the spectrum of the LSSB-AM modulated signal for $m(t)$

2. The power in the message signal is

$$P_m = \frac{1}{0.15}\int_0^{0.15} m^2(t)\,dt = 1.667$$

and therefore

$$P_u = \frac{A_c^2}{4} P_m = 0.416$$

3. The post-demodulation SNR is given by

$$10\log_{10}\left(\frac{P_R}{P_n}\right)_o = 10$$

Hence, $P_n = 0.1P_R = 0.1P_u = 0.0416$.

The MATLAB script for this problem follows.

M-FILE

```
% lssb.m
% Matlab demonstration script for LSSB-AM modulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                  % signal duration
ts=0.001;                                % sampling interval
fc=250;                                  % carrier frequency
snr=10;                                  % SNR in dB (logarithmic)
fs=1/ts;                                 % sampling frequency
df=0.25;                                 % desired freq. resolution
t=[0:ts:t0];                             % time vector
snr_lin=10^(snr/10);                     % SNR
% the message vector
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                       % carrier vector
udsb=m.*c;                               % DSB modulated signal
[UDSB,udssb,df1]=fftseq(udsb,ts,df);     % Fourier transform
UDSB=UDSB/fs;                            % scaling
f=[0:df1:df1*(length(udssb)-1)]-fs/2;    % frequency vector
n2=ceil(fc/df1);                         % location of carrier in freq. vector
% remove the upper sideband from DSB
UDSB(n2:length(UDSB)-n2)=zeros(size(UDSB(n2:length(UDSB)-n2)));
ULSSB=UDSB;                              % generate LSSB-AM spectrum
[M,m,df1]=fftseq(m,ts,df);               % Fourier transform
M=M/fs;                                  % scaling
u=real(ifft(ULSSB))*fs;                  % generate LSSB signal from spectrum
signal_power=spower(udsb(1:length(t)))/2;
%                                        % compute signal power
```

```
noise_power=signal_power/snr_lin;          % compute noise power
noise_std=sqrt(noise_power);               % compute noise standard deviation
noise=noise_std*randn(1,length(u));        % generate noise vector
r=u+noise;                                 % add the signal to noise
[R,r,df1]=fftseq(r,ts,df);                 % Fourier transform
R=R/fs;                                    % scaling
pause  % Press a key to show the modulated signal power
signal_power
pause  % Press any key to see a plot of the message signal
clf
subplot(2,1,1)
plot(t,m(1:length(t)))
axis([0,0.15,-2.1,2.1])
xlabel('Time')
title('The message signal')
pause  % Press any key to see a plot of the carrier
subplot(2,1,2)
plot(t,c(1:length(t)))
xlabel('Time')
title('The carrier')
pause  % Press any key to see a plot of the modulated signal and its spectrum
clf
subplot(2,1,1)
plot([0:ts:ts*(length(u)-1)/8],u(1:length(u)/8))
xlabel('Time')
title('The LSSB-AM modulated signal')
subplot(2,1,2)
plot(f,abs(fftshift(ULSSB)))
xlabel('Frequency')
title('Spectrum of the LSSB-AM modulated signal')
pause  % Press any key to see the spectra of the message and the modulated signals
clf
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Spectrum of the message signal')
subplot(2,1,2)
plot(f,abs(fftshift(ULSSB)))
xlabel('Frequency')
title('Spectrum of the LSSB-AM modulated signal')

pause  % Press any key to see a noise sample
subplot(2,1,1)
plot(t,noise(1:length(t)))
title('Noise sample')
xlabel('Time')
pause  % Press a key to see the modulated signal and noise
subplot(2,1,2)
plot(t,r(1:length(t)))
title('Modulated signal and noise')
xlabel('Time')
subplot(2,1,1)
pause % Press any key to see the spectrum of the modulated signal
plot(f,abs(fftshift(ULSSB)))
```

```
title('Modulated signal spectrum')
xlabel('Frequency')
subplot(2,1,2)


pause % Press a key to see the modulated signal noise in freq. domain
plot(f,abs(fftshift(R)))
title('Modulated signal noise spectrum')
xlabel('Frequency')
```

The m-files ussb_mod.m and lssb_mod.m given next modulate the message signal given in vector m using USSB and LSSB modulation schemes.

M-FILE

```
function u=ussb_mod(m,ts,fc)
%              u=ussb_mod(m,ts,fc)
%USSB_MOD      takes signal m sampled at ts and carrier
%              freq. fc as input and returns the USSB modulated
%              signal. ts << 1/2fc.
t=[0:length(m)-1]*ts;
u=m.*cos(2*pi*t)-imag(hilbert(m)).*sin(2*pi*t);
```

M-FILE

```
function u=lssb_mod(m,ts,fc)
%              u=lssb_mod(m,ts,fc)
%LSSB_MOD      takes signal m sampled at ts and carrier
%              freq. fc as input and returns the LSSB modulated
%              signal. ts << 1/2fc.
t=[0:length(m)-1]*ts;
u=m.*cos(2*pi*t)+imag(hilbert(m)).*sin(2*pi*t);
```

3.3 Demodulation of AM Signals

Demodulation is the process of extracting the message signal from the modulated signal. The demodulation process depends on the type of modulation employed. For DSB-AM and SSB-AM, the demodulation method is coherent demodulation, which requires the existence of a signal with the same frequency and phase of the carrier at the receiver. For conventional AM, envelope detectors are used for demodulation. In this case precise knowledge of the frequency and the phase of the carrier at the receiver is not crucial, so the demodulation process is much easier. Coherent demodulation for DSB-AM and SSB-AM

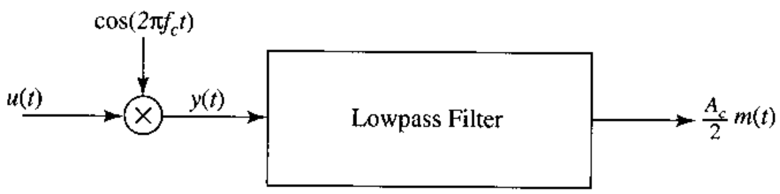


Figure 3.9 Demodulation of DSB-AM signals

consists of multiplying (mixing) the modulated signal by a sinusoidal with the same frequency and phase of the carrier and then passing the product through a lowpass filter. The oscillator that generates the required sinusoidal at the receiver is called the *local oscillator.*

3.3.1 DSB-AM Demodulation

In the DSB case the modulated signal is given by $A_c m(t)\cos(2\pi f_c t)$, which when multiplied by $\cos(2\pi f_c t)$, or mixed with $\cos(2\pi f_c t)$, results in

$$y(t) = A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t) = \frac{A_c}{2}m(t) + \frac{A_c}{2}m(t)\cos(4\pi f_c t) \tag{3.3.1}$$

where $y(t)$ denotes the mixer output, and its Fourier transform is given by

$$Y(f) = \frac{A_c}{2}M(f) + \frac{A_c}{4}M(f-2f_c) + \frac{A_c}{4}M(f+2f_c) \tag{3.3.2}$$

As it is seen, the mixer output has a lowpass component of $(A_c/2)M(f)$ and high-frequency components in the neighborhood of $\pm 2f_c$. When $y(t)$ passes through a lowpass filter with bandwidth W, the high-frequency components will be filtered out and the lowpass component, $(A_c/2)m(t)$, which is proportional to the message signal, will be demodulated. This process is shown in Figure 3.9.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.5 [DSB-AM demodulation] The message signal $m(t)$ is defined as

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

This message DSB-AM modulates the carrier $c(t) = \cos 2\pi f_c t$, and the resulting modulated signal is denoted by $u(t)$. It is assumed that $t_0 = 0.15$ s and $f_c = 250$ Hz.

1. Obtain the expression for $u(t)$.

2. Derive the spectra of $m(t)$ and $u(t)$.
3. Demodulate the modulated signal $u(t)$ and recover $m(t)$. Plot the results in the time and frequency domains.

SOLUTION

1, 2. The first two parts of this problem are the same as the first two parts of Illustrative Problem 3.1, and we repeat only those results here:

$$u(t) = \left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos(500\pi t)$$

and

$$\begin{aligned}\mathcal{F}[m(t)] &= \frac{t_0}{3}e^{-j\pi f t_0/3}\operatorname{sinc}\left(\frac{t_0 f}{3}\right) - \frac{2t_0}{3}e^{-j\pi f t_0}\operatorname{sinc}\left(\frac{t_0 f}{3}\right)\\ &= \frac{t_0}{3}e^{-j\pi f t_0/3}\operatorname{sinc}\left(\frac{t_0 f}{3}\right)\left(1 - 2e^{-j2\pi t_0 f/3}\right)\\ &= 0.05e^{-0.05j\pi f}\operatorname{sinc}(0.05f)\left(1 - 2e^{-0.01j\pi f}\right)\end{aligned}$$

Therefore,

$$\begin{aligned}U(f) = {}& 0.025e^{-0.05j\pi(f-250)}\operatorname{sinc}(0.05(f-250))\left(1 - 2e^{-0.1j\pi(f-250)}\right)\\ &+ 0.025e^{-0.05j\pi(f+250)}\operatorname{sinc}(0.05(f+250))\left(1 - 2e^{-0.1j\pi(f+250)}\right)\end{aligned}$$

3. To demodulate, we multiply $u(t)$ by $\cos(2\pi f_c t)$ to obtain the mixer output $y(t)$:

$$\begin{aligned}y(t) &= u(t)\cos(2\pi f_c t)\\ &= \left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos^2(500\pi t)\\ &= \frac{1}{2}\left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\\ &\quad + \frac{1}{2}\left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos(1000\pi t)\end{aligned}$$

whose Fourier transform is given by

$$\begin{aligned}Y(f) = {}& 0.025e^{-0.05j\pi f}\operatorname{sinc}(0.05f)\left(1 - 2e^{-0.01j\pi f}\right)\\ &+ 0.0125e^{-0.05j\pi(f-500)}\operatorname{sinc}\left(0.05(f-500)\right)\left(1 - 2e^{-0.1j\pi(f-500)}\right)\\ &+ 0.0125e^{-0.05j\pi(f+500)}\operatorname{sinc}\left(0.05(f+500)\right)\left(1 - 2e^{-0.1j\pi(f+500)}\right)\end{aligned}$$

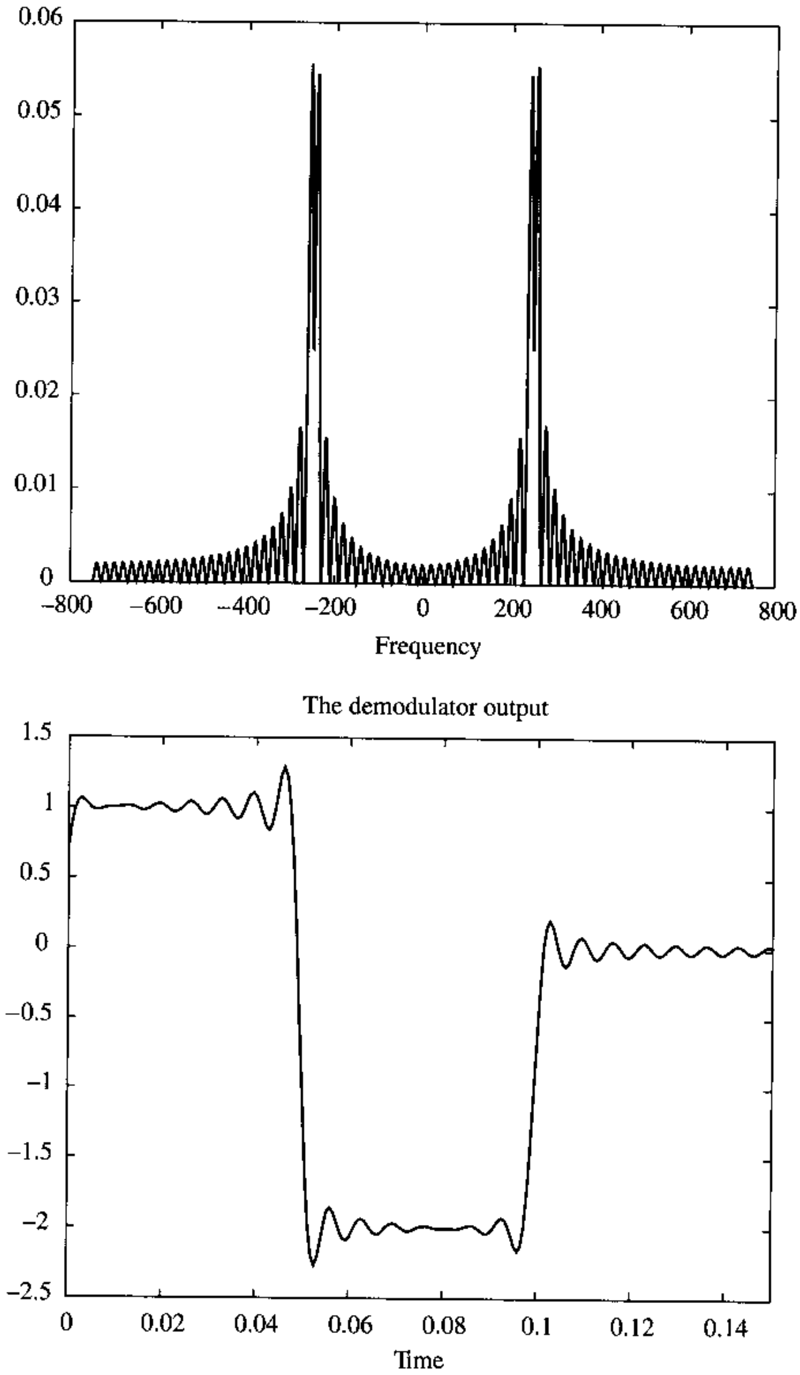


Figure 3.10 Spectra of the modulated signal and the mixer output in Illustrative Problem 3.5

where the first term corresponds to the message signal and the last two terms correspond to the high-frequency terms at twice the carrier frequency. We see that filtering the first term yields the original message signal (up to a proportionality constant). A plot of the magnitudes of $U(f)$ and $Y(f)$ is shown in Figure 3.10.

As shown, the spectrum of the mixer output has a lowpass component that is quite similar to the spectrum of the message signal, except for a factor of $\frac{1}{2}$, and a bandpass component located at $\pm 2f_c$ (in this case, 500 Hz). Using a lowpass filter, we can simply separate the lowpass component from the bandpass component. In order to recover the message signal $m(t)$, we pass $y(t)$ through a lowpass filter with a bandwidth of 150 Hz. The choice of the bandwidth of the filter is more or less arbitrary here because the message signal is not strictly bandlimited. For a strictly bandlimited message signal, the appropriate choice for the bandwidth of the lowpass filter would be W, the bandwidth of the message signal. Therefore, the ideal lowpass filter employed here has a characteristic

$$H(f) = \begin{cases} 1, & |f| \leq 150 \\ 0, & \text{otherwise} \end{cases}$$

A comparison of the spectra of $m(t)$ and the demodulator output is shown in Figure 3.11, and a comparison in the time domain is shown in Figure 3.12.

The MATLAB script for this problem follows.

M-FILE

```
% dsb_dem.m
% Matlab demonstration script for DSB-AM demodulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                 % signal duration
ts=1/1500;                              % sampling interval
fc=250;                                 % carrier frequency
fs=1/ts;                                % sampling frequency
t=[0:ts:t0];                            % time vector
df=0.3;                                 % desired frequency resolution
% message signal
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                      % carrier signal
u=m.*c;                                 % modulated signal
y=u.*c;                                 % mixing
[M,m,df1]=fftseq(m,ts,df);              % Fourier transform
M=M/fs;                                 % scaling
[U,u,df1]=fftseq(u,ts,df);              % Fourier transform
U=U/fs;                                 % scaling
[Y,y,df1]=fftseq(y,ts,df);              % Fourier transform
Y=Y/fs;                                 % scaling
f_cutoff=150;                           % cutoff freq. of the filter
n_cutoff=floor(150/df1);                % design the filter
f=[0:df1:df1*(length(y)-1)]-fs/2;
H=zeros(size(f));
H(1:n_cutoff)=2*ones(1,n_cutoff);
H(length(f)-n_cutoff+1:length(f))=2*ones(1,n_cutoff);
DEM=H.*Y;                               % spectrum of the filter output
dem=real(ifft(DEM))*fs;                 % filter output
pause % Press a key to see the effect of mixing
```

```
clf
subplot(3,1,1)
plot(f,fftshift(abs(M)))
title('Spectrum of the Message Signal')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(U)))
title('Spectrum of the Modulated Signal')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
pause % Press a key to see the effect of filtering on the mixer output
clf
subplot(3,1,1)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(H)))
title('Lowpass Filter Characteristics')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(DEM)))
title('Spectrum of the Demodulator Output')
xlabel('Frequency')
pause % Press a key to compare the spectra of the message and the received signal
clf
subplot(2,1,1)
plot(f,fftshift(abs(M)))
title('Spectrum of the Message Signal')
xlabel('Frequency')
subplot(2,1,2)
plot(f,fftshift(abs(DEM)))
title('Spectrum of the Demodulator Output')
xlabel('Frequency')
pause % Press a key to see the message and the demodulator output signals
subplot(2,1,1)
plot(t,m(1:length(t)))
title('The Message Signal')
xlabel('Time')
subplot(2,1,2)
plot(t,dem(1:length(t)))
title('The Demodulator Output')
xlabel('Time')
```

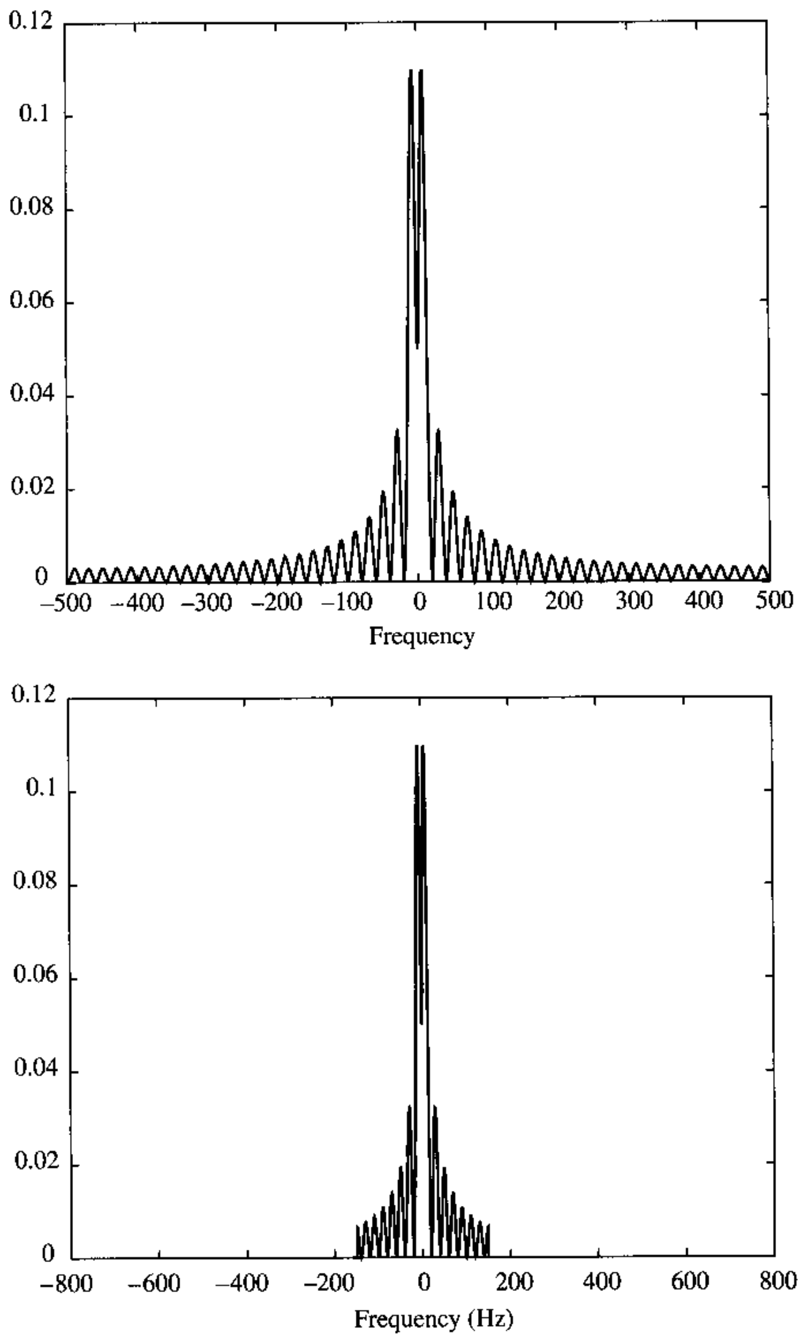


Figure 3.11 Spectra of the message and the demodulated signals in Illustrative Problem 3.5

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.6 [Effect of phase error on DSB-AM demodulation] In the demodulation of DSB-AM signals we assumed that the phase of the local oscillator is equal to the phase of the carrier. If that is not the case—that is, if there exists a phase shift ϕ between the local oscillator and the carrier—how would the demodulation process change?

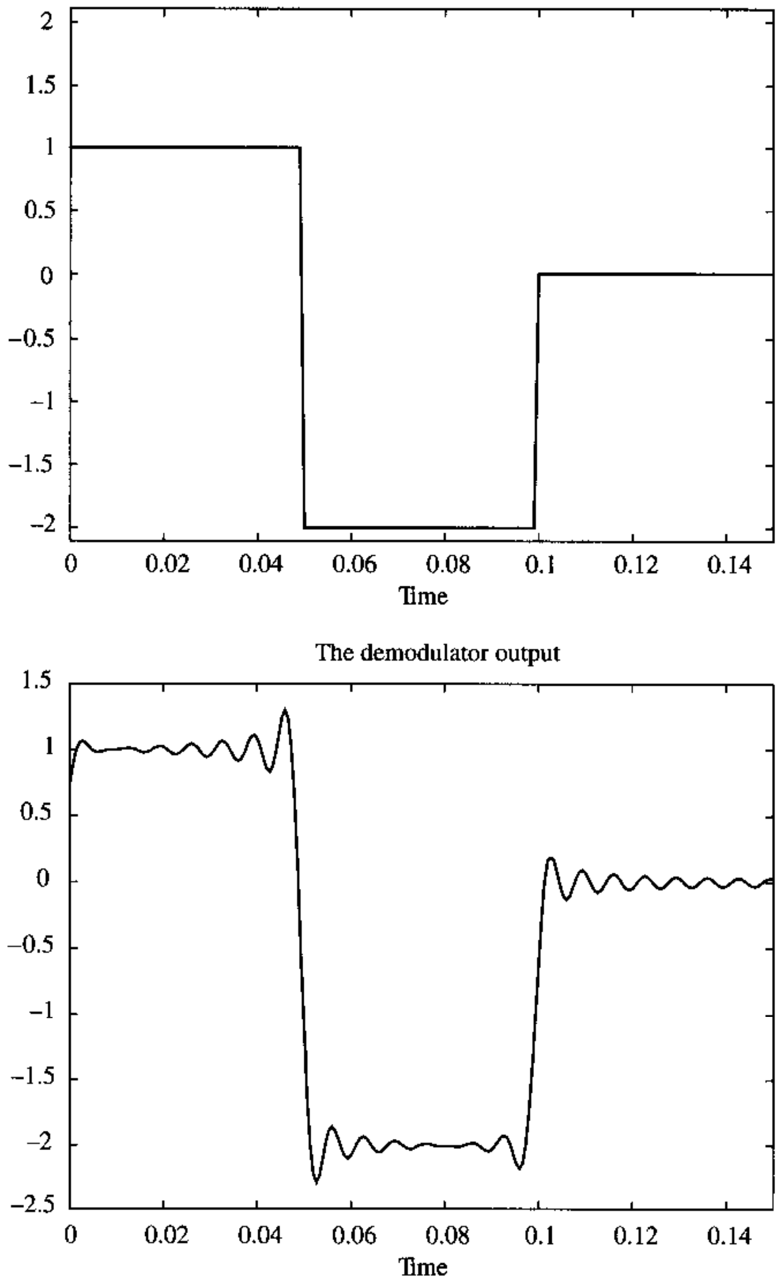


Figure 3.12 Message and demodulator output in Illustrative Problem 3.5

SOLUTION

In this case we have $u(t) = A_c m(t) \cos(2\pi f_c t)$, and the local oscillator generates a sinusoidal signal given by $\cos(2\pi f_c t + \phi)$. Mixing these two signals gives

$$y(t) = A_c m(t) \cos(2\pi f_c t) \times \cos(2\pi f_c t + \phi) \tag{3.3.3}$$

$$= \frac{A_c}{2} m(t) \cos(\phi) + \frac{A_c}{2} m(t) \cos(4\pi f_c t + \phi) \tag{3.3.4}$$

As before, there are two terms present in the mixer output. The bandpass term can be filtered out by a lowpass filter. The lowpass term, $(A_c/2)m(t)\cos(\phi)$, depends on ϕ, however. The power in the lowpass term is given by

$$P_{\text{dem}} = \frac{A_c^2}{4} P_m \cos^2\phi \tag{3.3.5}$$

where P_m denotes the power in the message signal. We can see, therefore, that in this case we can recover the message signal with essentially no distortion, but we will suffer a power loss of $\cos^2\phi$. For $\phi = \pi/4$ this power loss is 3 dB, and for $\phi = \pi/2$ nothing is recovered in the demodulation process.

3.3.2 SSB-AM Demodulation

The demodulation process of SSB-AM signals is basically the same as the demodulation process for DSB-AM signals—that is, mixing followed by lowpass filtering. In this case,

$$u(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t) \tag{3.3.6}$$

where the minus sign corresponds to the USSB and the plus sign corresponds to the LSSB. Mixing $u(t)$ with the local oscillator output, we obtain

$$\begin{aligned} y(t) &= \frac{A_c}{2} m(t)\cos^2(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t) \\ &= \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(4\pi f_c t) \mp \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t) \end{aligned} \tag{3.3.7}$$

which contains bandpass components at $\pm 2f_c$ and a lowpass component proportional to the message signal. The lowpass component can be filtered out using a lowpass filter to recover the message signal. This process for the USSB-AM case is depicted in Figure 3.13.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.7 [LSSB-AM example] In a USSB-AM modulation system, if the message signal is

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

with $t_0 = 0.15$ s, and the carrier has a frequency of 250 Hz, find $U(f)$ and $Y(f)$ and compare the demodulated signal with the message signal.

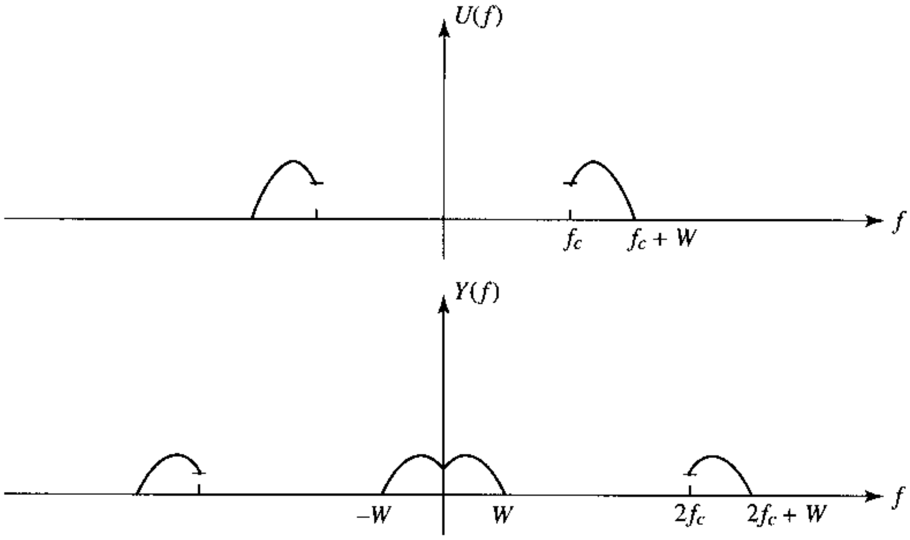


Figure 3.13 Demodulation of USSB-AM signals

SOLUTION

The modulated signal and its spectrum are given in Illustrative Problem 3.4. The expression for $U(f)$ is given by

$$U(f) = \begin{cases} 0.025e^{-0.05j\pi(f-250)}\text{sinc}(0.05(f-250))\left(1-2e^{-0.1j\pi(f-250)}\right) & \\ +\,0.025e^{-0.05j\pi(f+250)}\text{sinc}(0.05(f+250))\left(1-2e^{-0.1j\pi(f+250)}\right), & |f| \le f_c \\ 0, & \text{otherwise} \end{cases}$$

and

$$Y(f) = \frac{1}{2}U(f-f_c) + \frac{1}{2}U(f+f_c)$$

$$\approx \begin{cases} 0.0125e^{-0.05j\pi f}\text{sinc}(0.05f)\left(1-2e^{-0.01j\pi f}\right), & |f| \le f_c \\ 0.0125e^{-0.05j\pi(f-500)}\text{sinc}(0.05(f-500))\left(1-2e^{-0.01j\pi(f-500)}\right), & f_c \le f \le 2f_c \\ 0.0125e^{-0.05j\pi(f+500)}\text{sinc}(0.05(f+500))\left(1-2e^{-0.01j\pi(f+500)}\right), & -2f_c \le f \le -f_c \\ 0, & \text{otherwise} \end{cases}$$

A plot of $Y(f)$ is shown in Figure 3.14. The signal $y(t)$ is filtered by a lowpass filter with a cutoff frequency of 150 Hz; the spectrum of the output is shown in Figure 3.15. In Figure 3.16 the original message signal is compared with the demodulated signal.

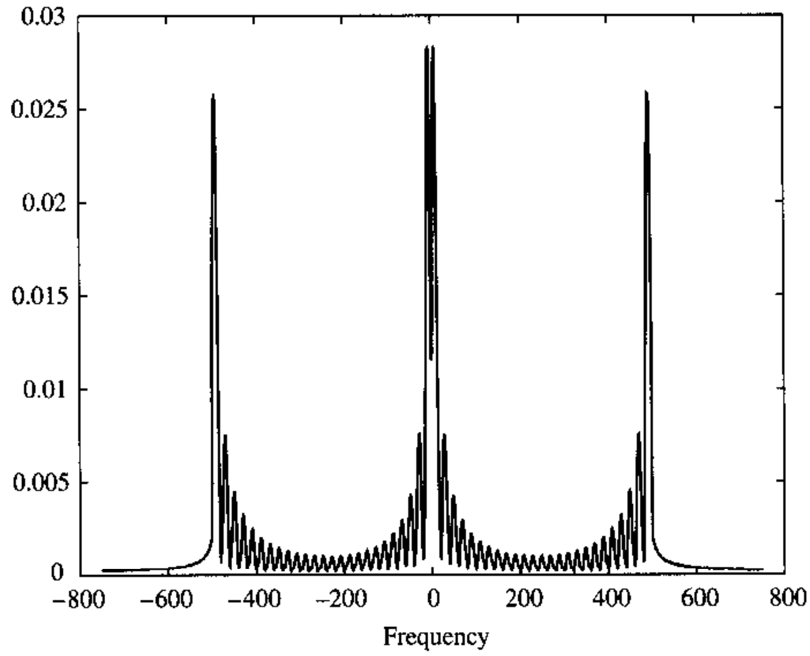


Figure 3.14 Magnitude spectrum of the mixer output in Illustrative Problem 3.7

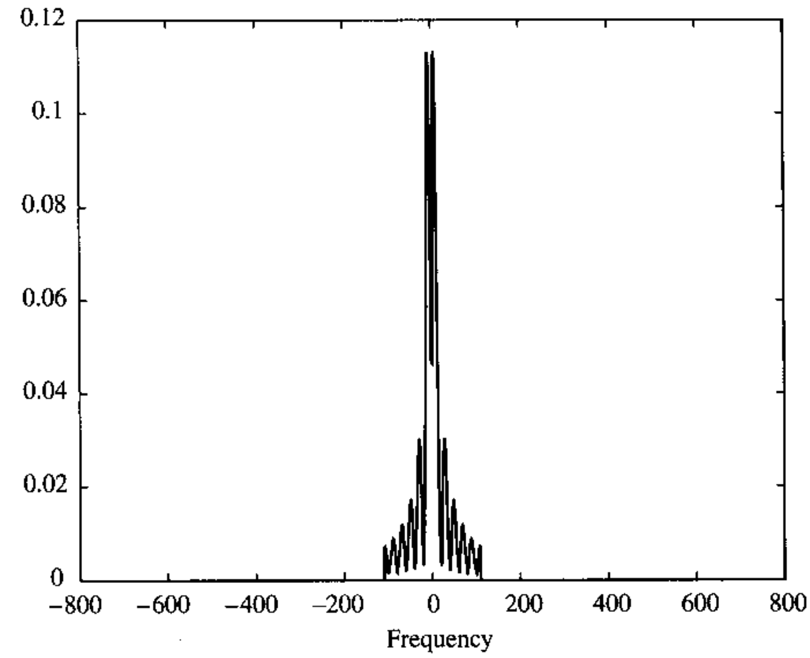


Figure 3.15 The demodulator output in Illustrative Problem 3.7

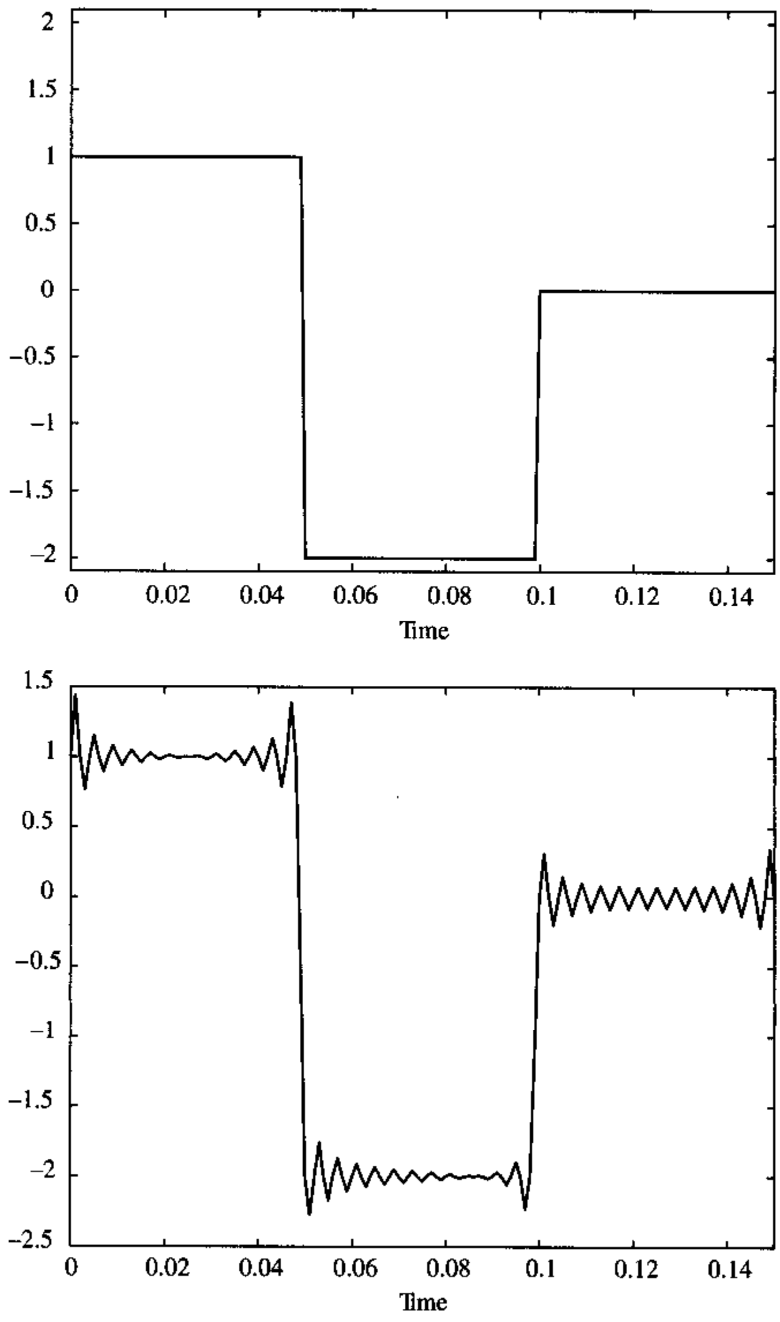


Figure 3.16 The message signal and the demodulator output in Illustrative Problem 3.7

The MATLAB script for this problem follows.

M-FILE

```
% lssb_dem.m
% Matlab demonstration script for LSSB-AM demodulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
```

```
t0=.15;                                % signal duration
ts=1/1500;                             % sampling interval
fc=250;                                % carrier frequency
fs=1/ts;                               % sampling frequency
df=0.25;                               % desired freq.resolution
t=[0:ts:t0];                           % time vector
% the message vector
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                     % carrier vector
udsb=m.*c;                             % DSB modulated signal
[UDSB,udsb,df1]=fftseq(udsb,ts,df);    % Fourier transform
UDSB=UDSB/fs;                          % scaling
n2=ceil(fc/df1);                       % location of carrier in freq. vector
% remove the upper sideband from DSB
UDSB(n2:length(UDSB)-n2)=zeros(size(UDSB(n2:length(UDSB)-n2)));
ULSSB=UDSB;                            % generate LSSB-AM spectrum
[M,m,df1]=fftseq(m,ts,df);             % spectrum of the message signal
M=M/fs;                                % scaling
f=[0:df1:df1*(length(M)-1)]-fs/2;      % frequency vector
u=real(ifft(ULSSB))*fs;                % generate LSSB signal from spectrum
% mixing
y=u.*cos(2*pi*fc*[0:ts:ts*(length(u)-1)]);
[Y,y,df1]=fftseq(y,ts,df);             % spectrum of the output of the mixer
Y=Y/fs;                                % scaling
f_cutoff=150;                          % choose the cutoff freq. of the filter
n_cutoff=floor(150/df);                % design the filter
H=zeros(size(f));
H(1:n_cutoff)=4*ones(1,n_cutoff);
% spectrum of the filter output
H(length(f)-n_cutoff+1:length(f))=4*ones(1,n_cutoff);
DEM=H.*Y;                              % spectrum of the filter output
dem=real(ifft(DEM))*fs;                % filter output
pause % Press a key to see the effect of mixing
clf
subplot(3,1,1)
plot(f,fftshift(abs(M)))
title('Spectrum of the Message Signal')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(ULSSB)))
title('Spectrum of the Modulated Signal')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
pause % Press a key to see the effect of filtering on the mixer output
clf
subplot(3,1,1)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(H)))
```

```
title('Lowpass Filter Characteristics')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(DEM)))
title('Spectrum of the Demodulator Output')
xlabel('Frequency')
pause % Press a key to see the message and the demodulator output signals
subplot(2,1,1)
plot(t,m(1:length(t)))
title('The Message Signal')
xlabel('Time')
subplot(2,1,2)
plot(t,dem(1:length(t)))
title('The Demodulator Output')
xlabel('Time')
```

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.8 [Effect of phase error on SSB-AM] What is the effect of phase error on SSB-AM?

SOLUTION

Assuming that the local oscillator generates a sinusoidal with a phase offset of ϕ with respect to the carrier, we have

$$\begin{aligned} y(t) &= u(t)\cos(2\pi f_c t + \phi) \\ &= \left[\frac{A_c}{2}m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t + \phi) \\ &= \frac{A_c}{4}m(t)\cos\phi \pm \frac{A_c}{4}\hat{m}(t)\sin\phi + \text{high-frequency terms} \end{aligned} \tag{3.3.8}$$

As seen, unlike the DSB-AM case, the effect of the phase offset here is not simply attenuating the demodulated signal. Here the demodulated signal is attenuated by a factor of $\cos\phi$ as well as distorted by addition of the $\pm(A_c/4)\hat{m}(t)\sin\phi$ term. In the special case of $\phi = \pi/2$, the Hilbert transform of the signal will be demodulated instead of the signal itself.

3.3.3 Conventional AM Demodulation

We have already seen that conventional AM is inferior to DSB-AM and SSB-AM when power and SNR are considered. The reason is that a usually large part of the modulated signal power is in the carrier component that does not carry information. The role of the

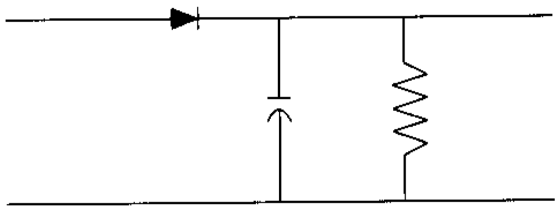


Figure 3.17 A simple envelope detector

carrier component is to make the demodulation of the conventional AM easier via envelope detection, as opposed to coherent demodulation required for DSB-AM and SSB-AM. Therefore, demodulation of AM signals is significantly less complex than the demodulation of DSB-AM and SSB-AM signals. Hence, this modulation scheme is widely used in broadcasting, where there exists a single transmitter and numerous receivers whose cost should be kept low. In envelope detection the envelope of the modulated signal is detected via a simple circuit consisting of a diode, a resistor, and a capacitor, as shown in Figure 3.17.

Mathematically, the envelope detector generates the envelope of the conventional AM signal, which is

$$V(t) = |1 + am_n(t)| \tag{3.3.9}$$

Because $1 + m_n(t) \geq 0$, we conclude that

$$V(t) = 1 + am_n(t) \tag{3.3.10}$$

where $m_n(t)$ is proportional to the message signal $m(t)$ and 1 corresponds to the carrier component that can be separated by a dc-block circuit. As seen in the preceding procedure, there is no need for knowledge of ϕ, the phase of the carrier signal. That is why such a demodulation scheme is called *noncoherent,* or *asynchronous, demodulation.* Recall from Chapter 1 that the envelope of a bandpass signal can be expressed as the magnitude of its lowpass equivalent signal. Thus, if $u(t)$ is the bandpass signal with central frequency f_c and the lowpass equivalent to $u(t)$ is denoted by $u_l(t)$, then the envelope of $u(t)$, denoted by $V(t)$, can be expressed as

$$\begin{aligned} V(t) &= \sqrt{u_{lr}^2(t) + u_{li}^2(t)} \\ &= \sqrt{u_c^2(t) + u_s^2(t)} \end{aligned} \tag{3.3.11}$$

where $u_c(t)$ and $u_s(t)$ represent the in-phase and the quadrature components of the bandpass signal $u(t)$. Therefore, in order to obtain the envelope, it is enough to obtain the lowpass equivalent of the bandpass signal. The envelope is simply the magnitude of the lowpass equivalent of the bandpass signal.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.9 [Envelope detection] The message signal

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

modulates the carrier $c(t) = \cos(2\pi f_c t)$ using a conventional AM modulation scheme. It is assumed that $f_c = 250$ Hz and $t_0 = 0.15$ s, and the modulation index is $a = 0.85$.

1. Using envelope detection, demodulate the message signal.

2. If the message signal is periodic with a period equal to t_0 and if an AWGN process is added to the modulated signal such that the power in the noise process is one-hundredth the power in the modulated signal, use an envelope demodulator to demodulate the received signal. Compare this case with the case where there is no noise present.

SOLUTION

1. As in Illustrative Problem 3.3, we have

$$\begin{aligned} u(t) &= \left[1 + 0.85 \frac{m(t)}{2}\right] \cos(2\pi f_c t) \\ &= \left[1 + 0.425\Pi\left(\frac{t - 0.025}{0.05}\right) - 0.85\Pi\left(\frac{t - 0.075}{0.05}\right)\right] \cos(500\pi t) \end{aligned}$$

If an envelope detector is used to demodulate the signal and the carrier component is removed by a dc-block, then the original message $m(t)$ is recovered. Note that a crucial point in the recovery of $m(t)$ is that for all values of t, the expression $1 + am_n(t)$ is positive; therefore, the envelope of the signal $[1 + am_n(t)] \cos(2\pi f_c t)$, which is $V(t) = |1 + am_n(t)|$, is equal to $1 + am_n(t)$, from which $m(t)$ can be recovered easily. Plots of the conventional AM modulated signal and its envelope as detected by the envelope detector are shown in Figure 3.18.

After the envelope detector separates the envelope of the modulated signal, the dc component of the signal is removed and the signal is scaled to generate the demodulator output. Plots of the original message signal and the demodulator output are shown in Figure 3.19.

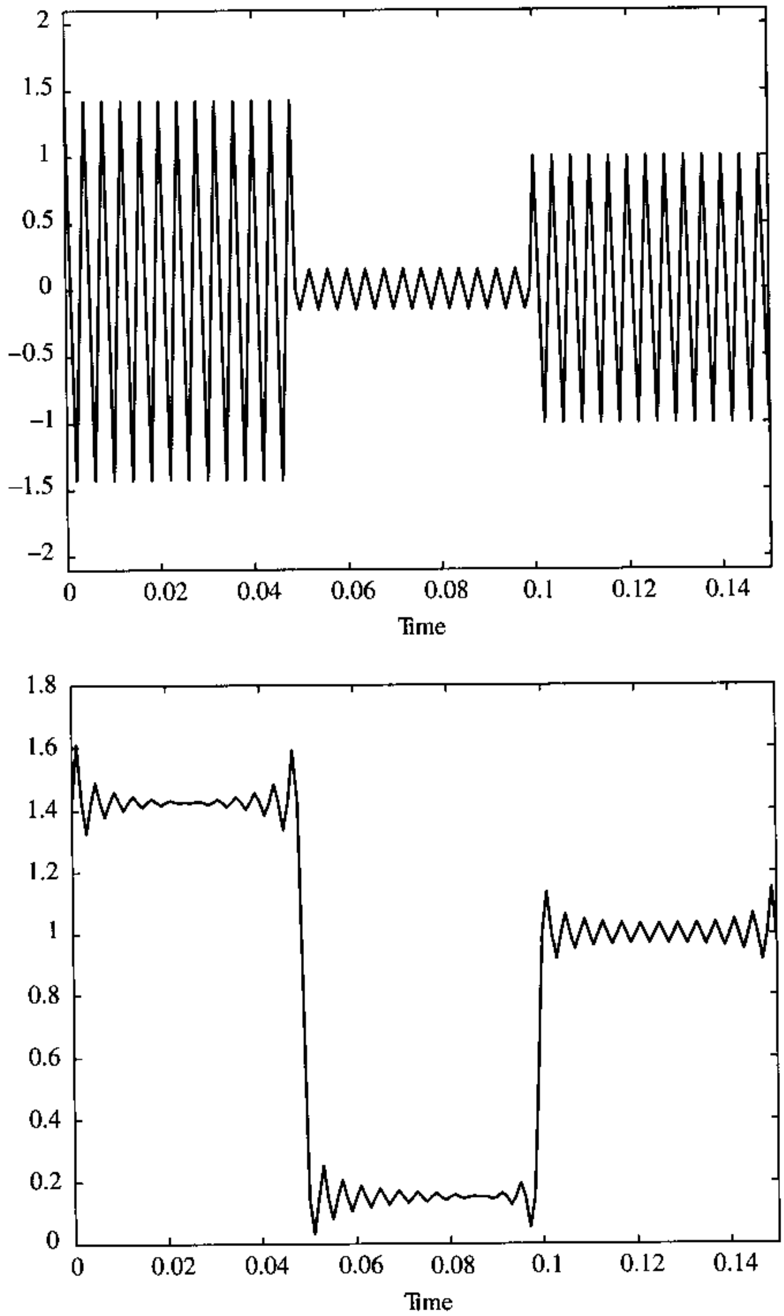


Figure 3.18 Conventional AM modulated signal and its envelope

2. When noise is present, there will also be some distortion due to the noise. In Figure 3.20 the received signal and its envelope are shown. In Figure 3.21 the message signal and the demodulated signal are compared for this case.

The MATLAB script for this problem follows.

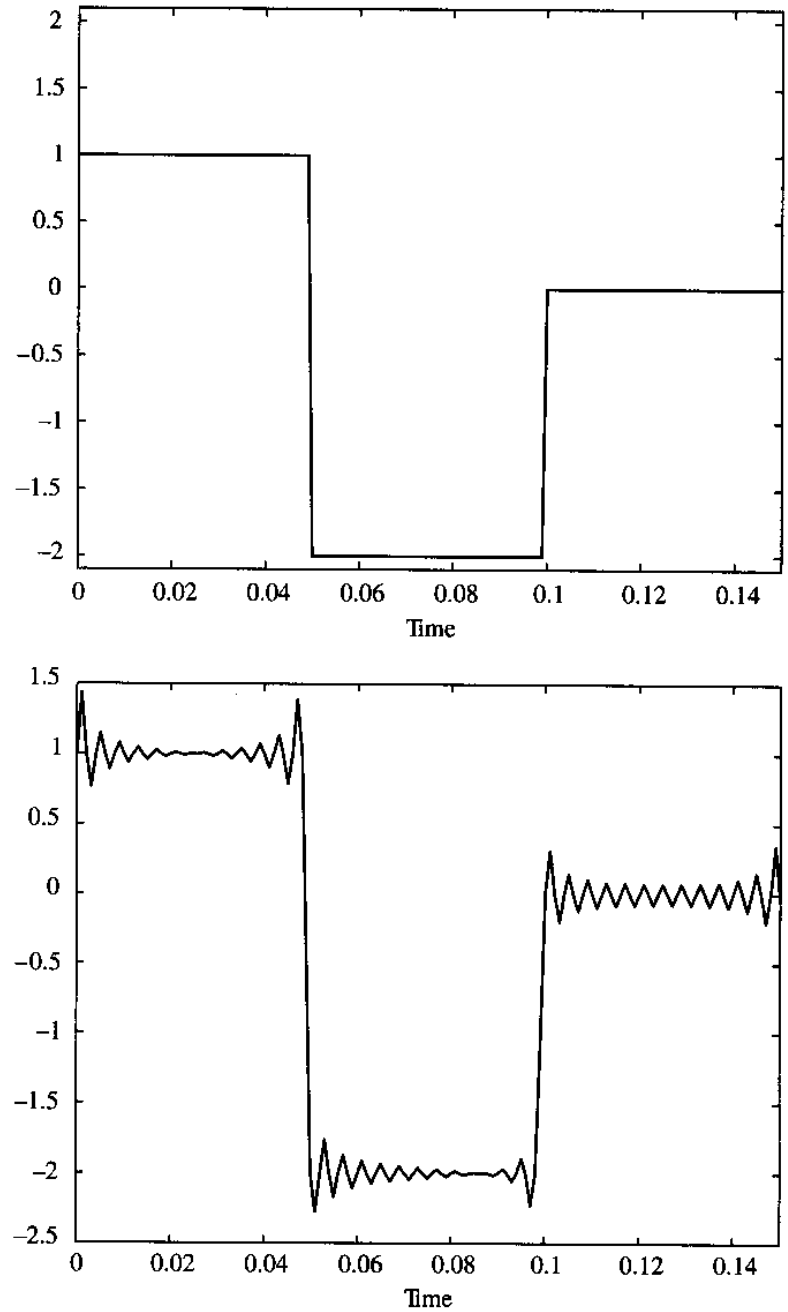
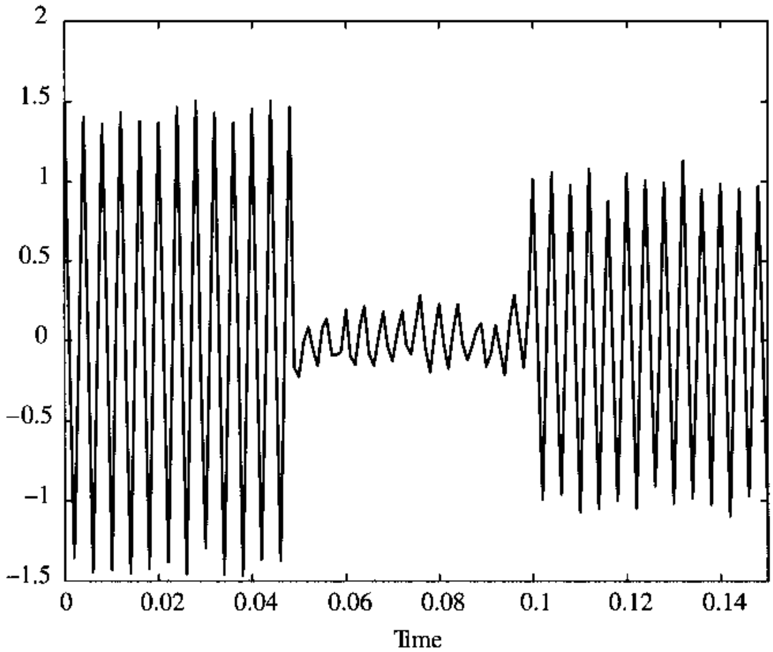


Figure 3.19 The message signal and the demodulated signal when no noise is present

M-FILE

```
% am-dem.m
% Matlab demonstration script for envelope detection. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                % signal duration
ts=0.001;                              % sampling interval
```



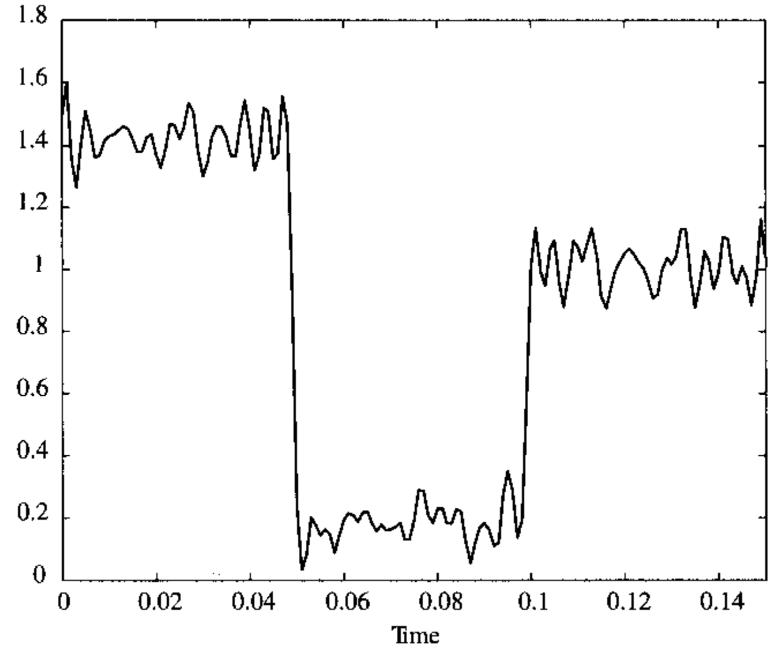


Figure 3.20 The received signal and its envelope in the presence of noise

```
fc=250;                                   % carrier frequency
a=0.85;                                   % modulation index
fs=1/ts;                                  % sampling frequency
t=[0:ts:t0];                              % time vector
df=0.25;                                  % required frequency resolution
% message signal
m=[ones(1,t0/(3*ts)),−2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                        % carrier signal
```

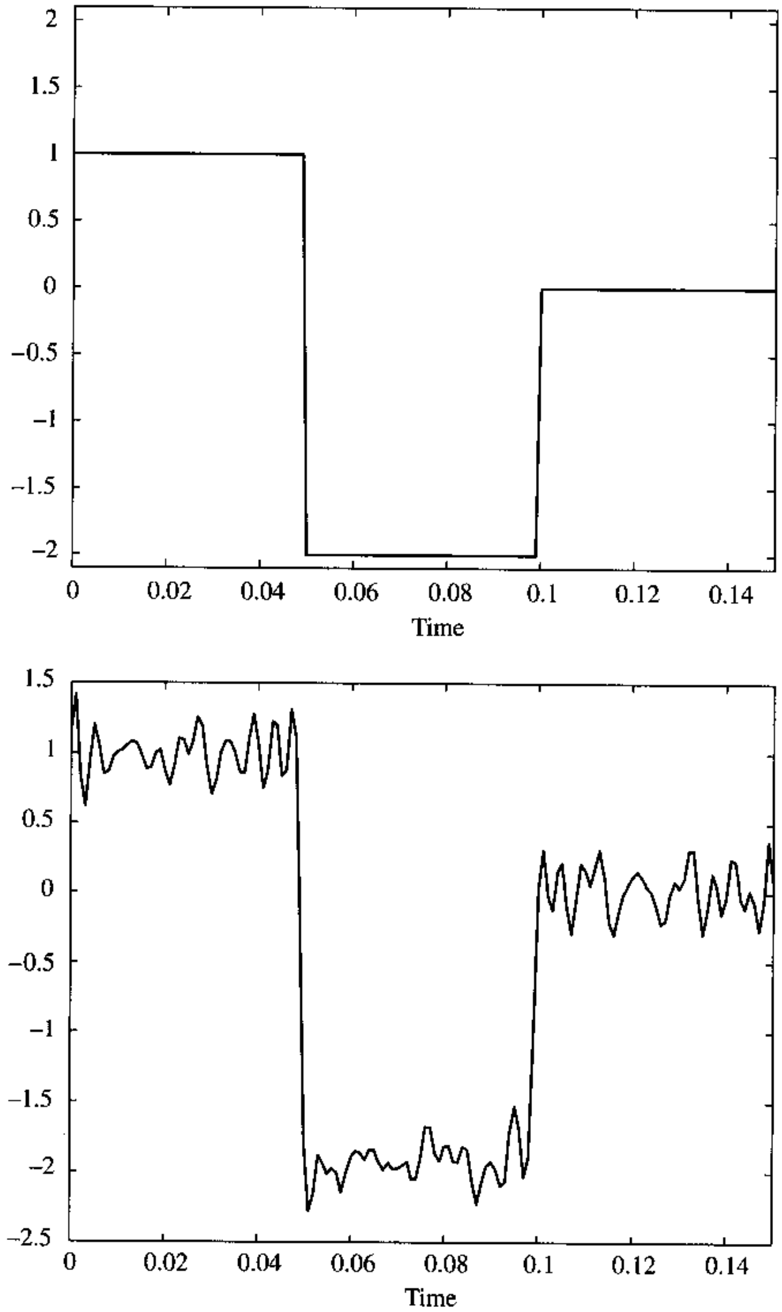


Figure 3.21 The message signal and the demodulated signal in the presence of noise

```
m_n=m/max(abs(m));                      % normalized message signal
[M,m,df1]=fftseq(m,ts,df);              % Fourier transform
f=[0:df1:df1*(length(m)-1)]-fs/2;       % frequency vector
u=(1+a*m_n).*c;                         % modulated signal
[U,u,df1]=fftseq(u,ts,df);              % Fourier transform
env=env_phas(u);                        % find the envelope
dem1=2*(env-1)/a;                       % remove dc and rescale
signal_power=spower(u(1:length(t)));    % power in modulated signal
noise_power=signal_power/100;           % noise power
```

```
noise_std=sqrt(noise_power);              % noise standard deviation
noise=noise_std*randn(1,length(u));       % generate noise
r=u+noise;                                % add noise to the modulated signal
[R,r,df1]=fftseq(r,ts,df);                % Fourier transform
env_r=env_phas(r);                        % envelope, when noise is present
dem2=2*(env_r-1)/a;                       % demodulate in the presence of noise
pause  % Press any key to see a plot of the message
subplot(2,1,1)
plot(t,m(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The message signal')
pause  % Press any key to see a plot of the modulated signal
subplot(2,1,2)
plot(t,u(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The modulated signal')
pause  % Press a key to see the envelope of the modulated signal
clf
subplot(2,1,1)
plot(t,u(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The modulated signal')
subplot(2,1,2)
plot(t,env(1:length(t)))
xlabel('Time')
title('Envelope of the modulated signal')
pause  % Press a key to compare the message and the demodulated signal
clf
subplot(2,1,1)
plot(t,m(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The message signal')
subplot(2,1,2)
plot(t,dem1(1:length(t)))
xlabel('Time')
title('The demodulated signal')
pause  % Press a key to compare in the presence of noise
clf
subplot(2,1,1)
plot(t,m(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The message signal')
subplot(2,1,2)
plot(t,dem2(1:length(t)))
xlabel('Time')
title('The demodulated signal in the presence of noise')
```

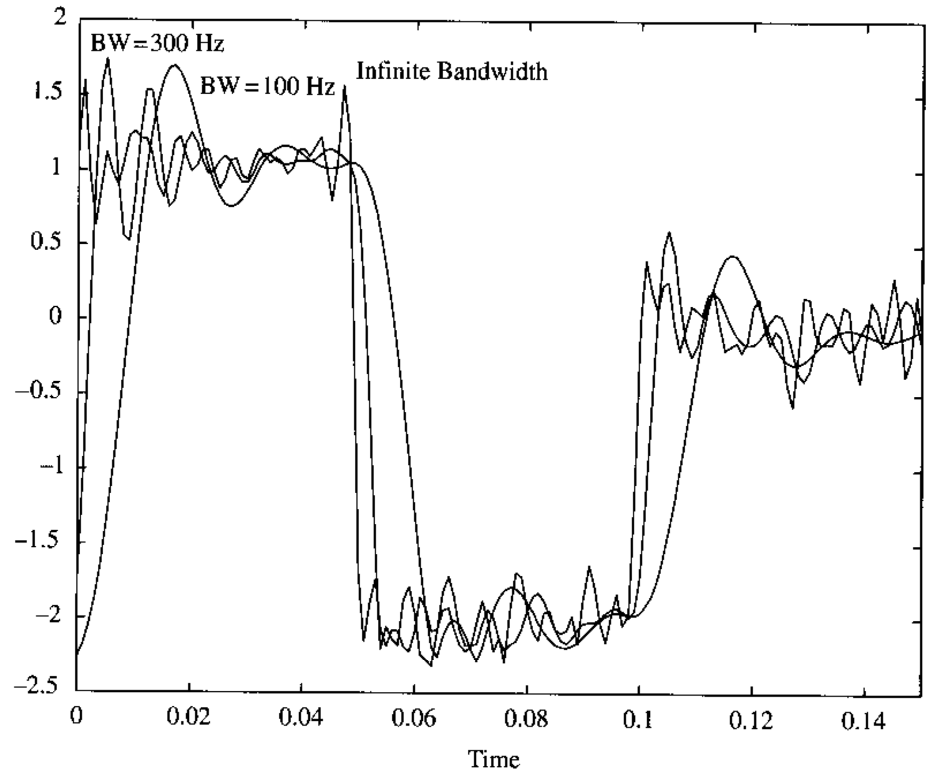


Figure 3.22 Effect of the bandwidth of the noise-limiting filter on the output of the envelope detector

COMMENT

In the demodulation process above, we have neglected the effect of the noise-limiting filter, which is a bandpass filter in the first stage of any receiver. In practice the received signal $r(t)$ is passed through the noise-limiting filter and then supplied to the envelope detector. In the preceding example, since the message bandwidth is not finite, passing $r(t)$ through any bandpass filter will cause distortion on the demodulated message, but it will also decrease the amount of noise in the demodulator output. In Figure 3.22 we have plotted the demodulator outputs when noise-limiting filters of different bandwidths are used. The case of infinite bandwidth is equivalent to the result shown in Figure 3.21.

3.4 Angle Modulation

Angle-modulation schemes, which include frequency modulation (FM) and phase modulation (PM), belong to the class of nonlinear modulation schemes. This family of modulation schemes is characterized by their high-bandwidth requirements and good performance in the presence of noise. These schemes can be visualized as modulation techniques that trade off bandwidth for power and, therefore, are used in situations where bandwidth is not the major concern and a high SNR is required. Frequency modulation is widely used in high-fidelity FM broadcasting, TV audio broadcasting, microwave carrier modulation, and point-to-point communication systems.

In our treatment of angle-modulation schemes, we again concentrate on their five basic properties—namely, time-domain representation, frequency-domain representation, bandwidth, power content, and, finally, SNR. Since there is a close relationship between PM and FM, we will treat them in parallel, with emphasis on FM.

The time-domain representation of angle-modulated signals, when the carrier is $c(t) = A_c \cos(2\pi f_c t)$ and the message signal is $m(t)$, is given by

$$u(t) = \begin{cases} A_c \cos\left(2\pi f_c t + k_p m(t)\right), & \text{PM} \\ A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right), & \text{FM} \end{cases} \tag{3.4.1}$$

where k_f and k_p represent the *deviation constants* of FM and PM, respectively. The frequency-domain representation of angle-modulated signals is, in general, very complex due to the nonlinearity of these modulation schemes. We treat only the case where the message signal $m(t)$ is a sinusoidal signal. We assume $m(t) = a\cos(2\pi f_m t)$ for PM and $m(t) = -a\sin(2\pi f_m t)$ for FM. Then the modulated signal is of the form

$$u(t) = \begin{cases} A_c \cos(2\pi f_c t + \beta_p \cos(2\pi f_m t)), & \text{PM} \\ A_c \cos(2\pi f_c t + \beta_f \cos(2\pi f_m t)), & \text{FM} \end{cases} \tag{3.4.2}$$

where

$$\begin{cases} \beta_p = k_p a \\ \beta_f = \dfrac{k_f a}{f_m} \end{cases} \tag{3.4.3}$$

and β_p and β_f are the *modulation indices* of PM and FM, respectively. In general, for a nonsinusoidal $m(t)$, the modulation indices are defined as

$$\begin{cases} \beta_p = k_p \max|m(t)| \\ \beta_f = \dfrac{k_f \max|m(t)|}{W} \end{cases} \tag{3.4.4}$$

where W is the bandwidth of the message signal $m(t)$. In the case of a sinusoidal message signal, the modulated signal can be represented by

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + n f_m)t) \tag{3.4.5}$$

where $J_n(\beta)$ is the Bessel function of the first kind and of order n and β is either β_p or β_f, depending on whether we are dealing with PM or FM. In the frequency domain we have

$$U(f) = \sum_{n=-\infty}^{\infty} \left[\frac{A_c J_n(\beta)}{2}\delta(f - (f_c + n f_m)) + \frac{A_c J_n(\beta)}{2}\delta(f + (f_c + n f_m))\right] \tag{3.4.6}$$

Obviously, the bandwidth of the modulated signal is not finite. However, we can define the *effective bandwidth* of the signal as the bandwidth containing 98% to 99% of the modulated signal power. This bandwidth is given by *Carson's rule* as

$$B_T = 2(\beta + 1)W \tag{3.4.7}$$

where β is the modulation index, W is the bandwidth of the message, and B_T is the bandwidth of the modulated signal.

The expression for the power content of the angle-modulated signals is very simple. Since the modulated signal is sinusoidal, with varying instantaneous frequency and constant amplitude, its power is constant and does not depend on the message signal. The power content for both FM and PM is given by

$$P_u = \frac{A_c^2}{2} \tag{3.4.8}$$

The SNR for angle-modulated signals, when no pre-emphasis and de-emphasis filtering is employed, is given by

$$\left(\frac{S}{N}\right)_o = \begin{cases} \frac{P_M \beta_p^2}{(\max|m(t)|)^2} \frac{P_R}{N_0 W}, & \text{PM} \\ 3\frac{P_M \beta_f^2}{(\max|m(t)|)^2} \frac{P_R}{N_0 W}, & \text{FM} \end{cases} \tag{3.4.9}$$

Since $\max|m(t)|$ denotes the maximum magnitude of the message signal, we can interpret $P_M/(\max|m(t)|)^2$ as the power in the *normalized message signal* and denote it by P_{M_n}. When pre-emphasis and de-emphasis filters with a 3-dB cutoff frequency equal to f_0 are employed, the SNR for FM is given by

$$\left(\frac{S}{N}\right)_{oPD} = \frac{(W/f_0)^3}{3\,[W/f_0 - \arctan(W/f_0)]} \left(\frac{S}{N}\right)_o \tag{3.4.10}$$

where $(S/N)_o$ is the SNR without pre-emphasis and de-emphasis filtering given by Equation (3.4.9).

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.10 [Frequency modulation] The message signal

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

modulates the carrier $c(t) = \cos(2\pi f_c t)$ using a frequency-modulation scheme. It is assumed that $f_c = 200$ Hz and $t_0 = 0.15$ s; the deviation constant is $k_f = 50$.

1. Plot the modulated signal.

2. Determine the spectra of the message and the modulated signals.

SOLUTION

1. We have

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right)$$

We have to find $\int_{-\infty}^{t} m(\tau)\, d\tau$. This can be done numerically or analytically, and the results are shown in Figure 3.23. Using the relation for $u(t)$ and the value of the integral of $m(t)$, as shown above, we obtain the expression for $u(t)$. A plot of $m(t)$ and $u(t)$ is shown in Figure 3.24.

2. Using MATLAB's Fourier transform routines, we obtain the expression for the spectrum of $u(t)$ shown in Figure 3.25. It is readily seen that unlike AM, in the FM case there does not exist a clear similarity between the spectrum of the message and the spectrum of the modulated signal. In this particular example the bandwidth of the message signal is not finite, and therefore to define the index of modulation, an approximate bandwidth for the message should be used in the expression

$$\beta = \frac{k_f \max |m(t)|}{W} \tag{3.4.11}$$

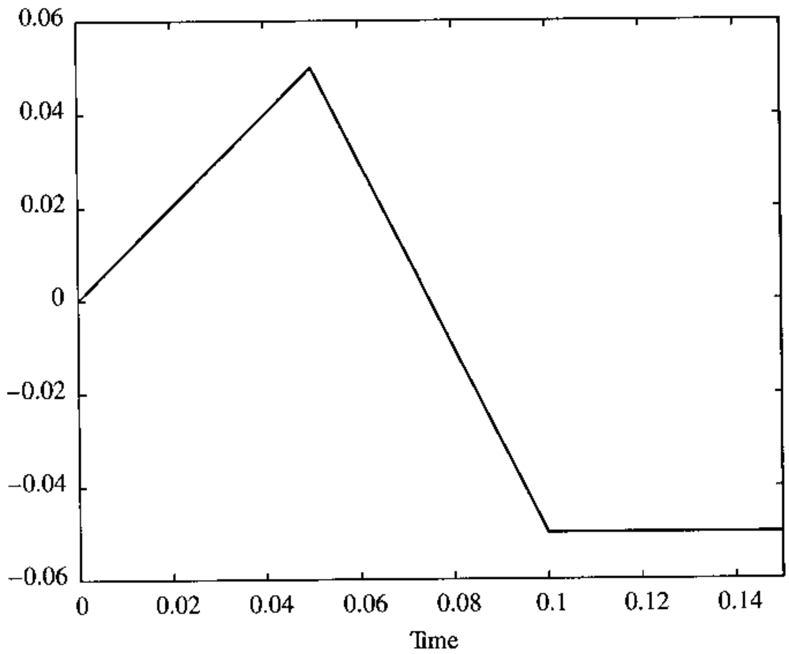


Figure 3.23 The integral of the message signal

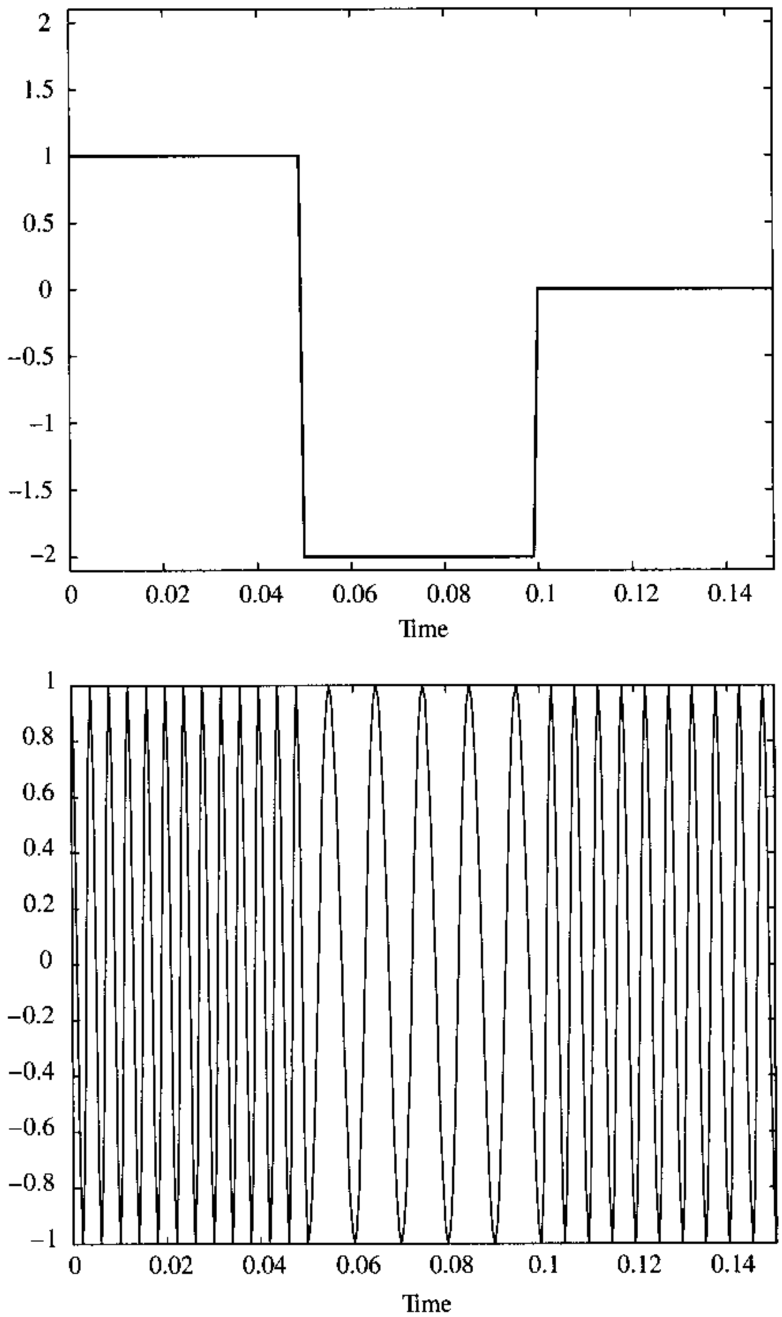


Figure 3.24 The message signal and the modulated signal

We can, for example, define the bandwidth as the width of the main lobe of the spectrum of $m(t)$, which results in

$$W = 20 \text{ Hz}$$

and so

$$\beta = \frac{50 \times 2}{20} = 10$$

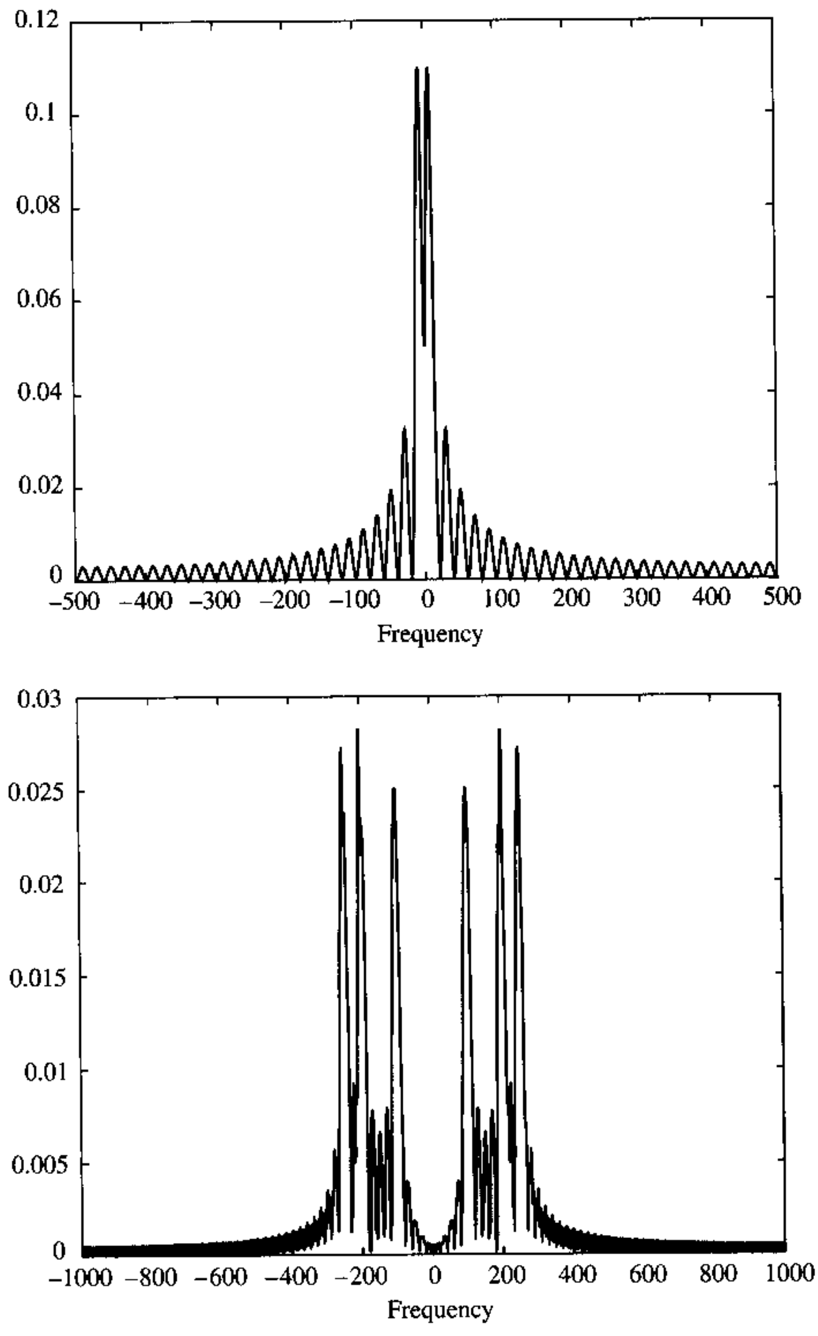


Figure 3.25 The magnitude spectra of the message and the modulated signal

The MATLAB script for this problem follows.

M-FILE

```
% fm1.m
% Matlab demonstration script for frequency modulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                              % signal duration
ts=0.0005;                           % sampling interval
fc=200;                              % carrier frequency
kf=50;                               % modulation index
fs=1/ts;                             % sampling frequency
t=[0:ts:t0];                         % time vector
df=0.25;                             % required frequency resolution
% message signal
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
int_m(1)=0;
for i=1:length(t)-1                  % integral of m
  int_m(i+1)=int_m(i)+m(i)*ts;
  echo off ;
end
echo on ;
[M,m,df1]=fftseq(m,ts,df);           % Fourier transform
M=M/fs;                              % scaling
f=[0:df1:df1*(length(m)-1)]-fs/2;    % frequency vector
u=cos(2*pi*fc*t+2*pi*kf*int_m);      % modulated signal
[U,u,df1]=fftseq(u,ts,df);           % Fourier transform
U=U/fs;                              % scaling
pause % Press any key to see plot of the message and the modulated signal
subplot(2,1,1)
plot(t,m(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The message signal')
subplot(2,1,2)
plot(t,u(1:length(t)))
axis([0 0.15 -2.1 2.1])
xlabel('Time')
title('The modulated signal')
pause   % Press any key to see plots of the magnitude of the message and the
        % modulated signal in the frequency domain.
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Magnitude spectrum of the message signal')
subplot(2,1,2)
plot(f,abs(fftshift(U)))
title('Magnitude spectrum of the modulated signal')
xlabel('Frequency')
```

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.11 [Frequency modulation] Let the message signal be

$$m(t) = \begin{cases} \text{sinc}(100t), & |t| \leq t_0 \\ 0, & \text{otherwise} \end{cases}$$

where $t_0 = 0.1$. This message modulates the carrier $c(t) = \cos(2\pi f_c t)$, where $f_c =$ 250 Hz. The deviation constant is $k_f = 100$.

1. Plot the modulated signal in the time and frequency domain.
2. Compare the demodulator output and the original message signal.

SOLUTION

1. We first integrate the message signal and then use the relation

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right)$$

to find $u(t)$. A plot of $u(t)$ together with the message signal is shown in Figure 3.26. The integral of the message signal is shown in Figure 3.27. A plot of the modulated signal in the frequency domain is shown in Figure 3.28.

2. To demodulate the FM signal, we first find the phase of the modulated signal $u(t)$. This phase is $2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau$, which can be differentiated and divided by $2\pi k_f$ to obtain $m(t)$. Note that in order to restore the phase and undo the effect of 2π phase foldings, we employ the unwrap.m function of MATLAB. Plots of the message signal and the demodulated signal are shown in Figure 3.29. As you can see, the demodulated signal is quite similar to the message signal.

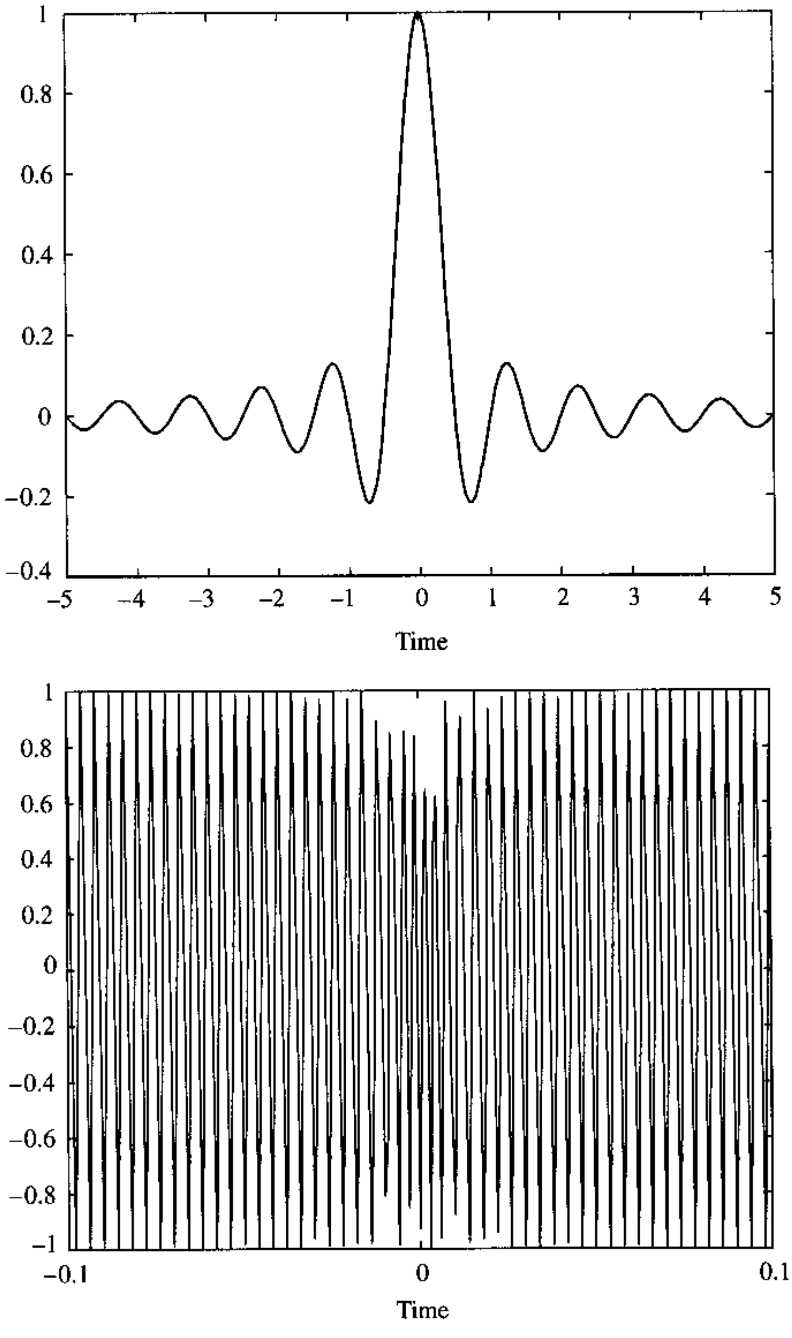


Figure 3.26 The message and the modulated signals

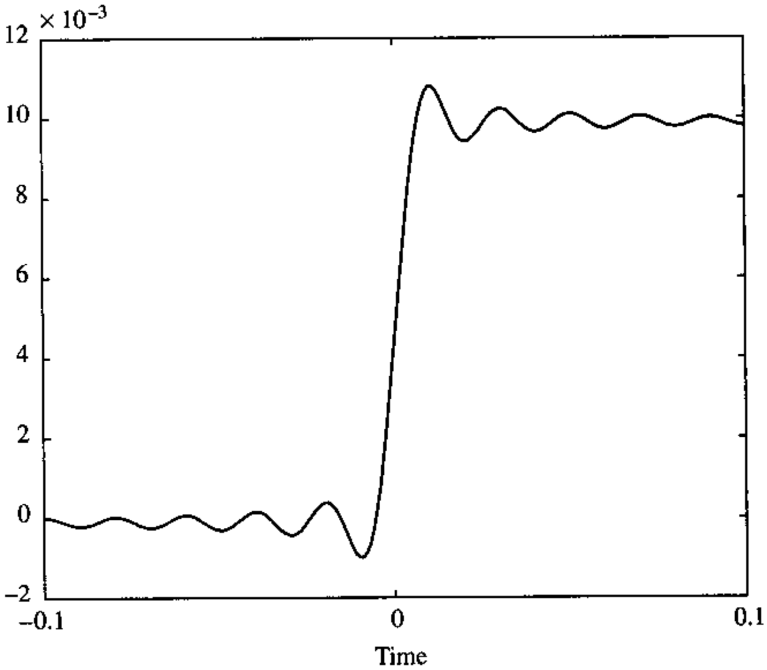


Figure 3.27 Integral of the message signal

The MATLAB script for this problem follows.

M-FILE

```
% fm2.m
% Matlab demonstration script for frequency modulation. The message signal
% is m(t)=sinc(100t).
echo on
t0=.2;                          % signal duration
ts=0.001;                       % sampling interval
fc=250;                         % carrier frequency
snr=20;                         % SNR in dB (logarithmic)
fs=1/ts;                        % sampling frequency
df=0.3;                         % required freq. resolution
t=[-t0/2:ts:t0/2];              % time vector
kf=100;                         % deviation constant
df=0.25;                        % required frequency resolution
m=sinc(100*t);                  % the message signal
int_m(1)=0;
for i=1:length(t)-1             % integral of m
  int_m(i+1)=int_m(i)+m(i)*ts;
  echo off ;
end
```

```
echo on ;
[M,m,df1]=fftseq(m,ts,df);                % Fourier transform
M=M/fs;                                   % scaling
f=[0:df1:df1*(length(m)-1)]-fs/2;         % frequency vector
u=cos(2*pi*fc*t+2*pi*kf*int_m);           % modulated signal
[U,u,df1]=fftseq(u,ts,df);                % Fourier transform
U=U/fs;                                   % scaling
[v,phase]=env_phas(u,ts,250);             % demodulation, find phase of u
phi=unwrap(phase);                        % restore original phase
dem=(1/(2*pi*kf))*(diff(phi)/ts);         % demodulator output, differentiate and scale phase
pause  % Press any key to see a plot of the message and the modulated signal
subplot(2,1,1)
plot(t,m(1:length(t)))
xlabel('Time')
title('The message signal')
subplot(2,1,2)
plot(t,u(1:length(t)))
xlabel('Time')
title('The modulated signal')
pause   % Press any key to see plots of the magnitude of the message and the
        % modulated signal in the frequency domain.
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Magnitude spectrum of the message signal')
subplot(2,1,2)
plot(f,abs(fftshift(U)))
title('Magnitude spectrum of the modulated signal')
xlabel('Frequency')
pause   % Press any key to see plots of the message and the demodulator output with no
        % noise
subplot(2,1,1)
plot(t,m(1:length(t)))
xlabel('Time')
title('The message signal')
subplot(2,1,2)
plot(t,dem(1:length(t)))
xlabel('Time')
title('The demodulated signal')
```

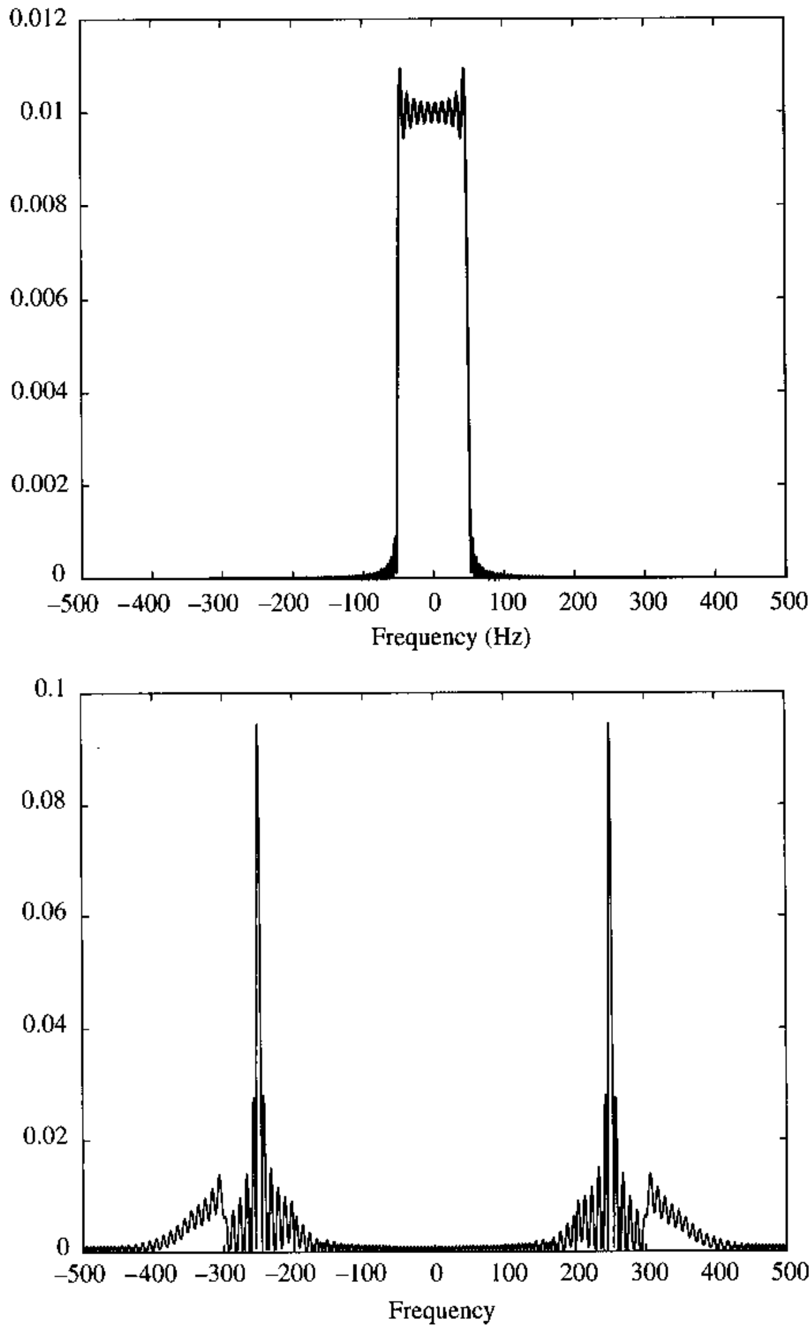


Figure 3.28 Magnitude spectra of the message and the modulated signal

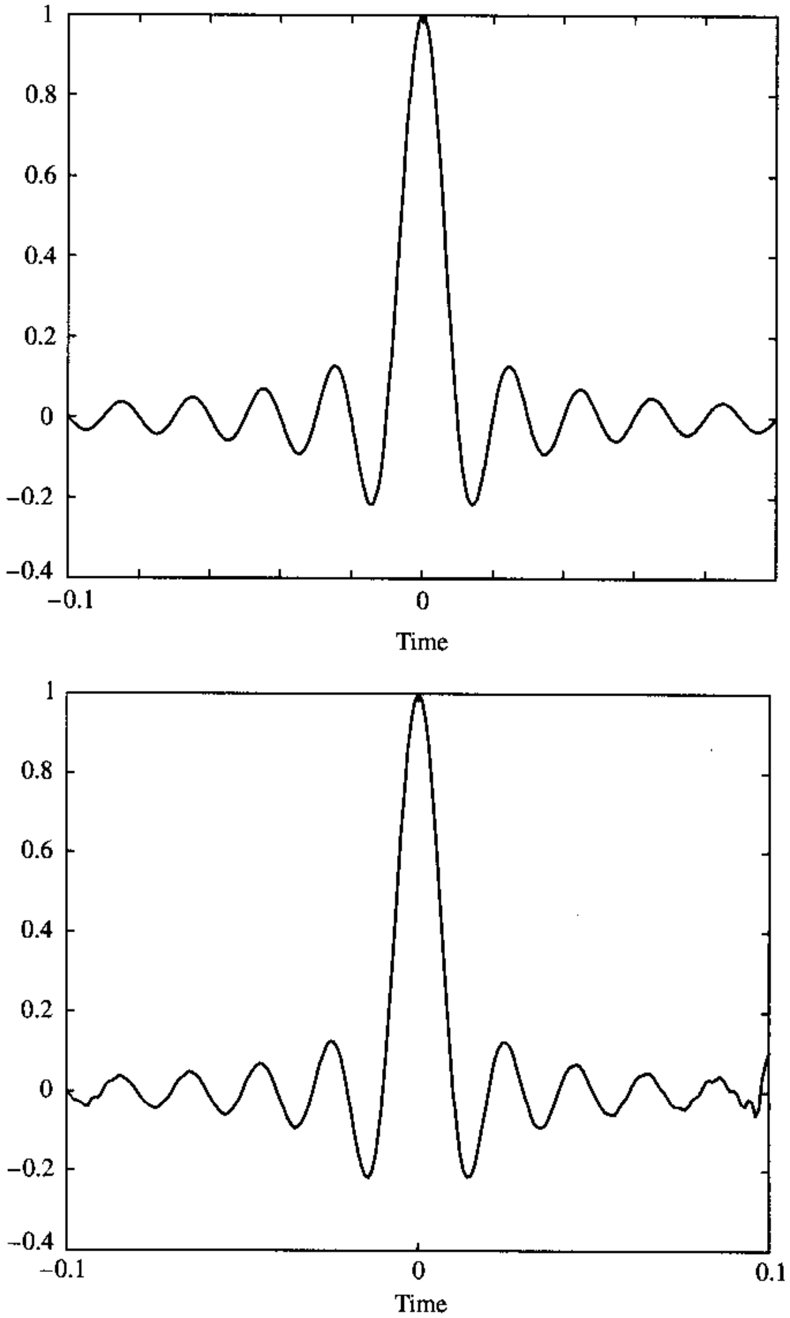


Figure 3.29 The message signal and the demodulated signal